



Computational Learning Theory

Frequent Substring Mining

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Why sequences in ML?(1)

- Sentences are strings(sequences) consisting of characters in an alphabet.



[Wikipedia]

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

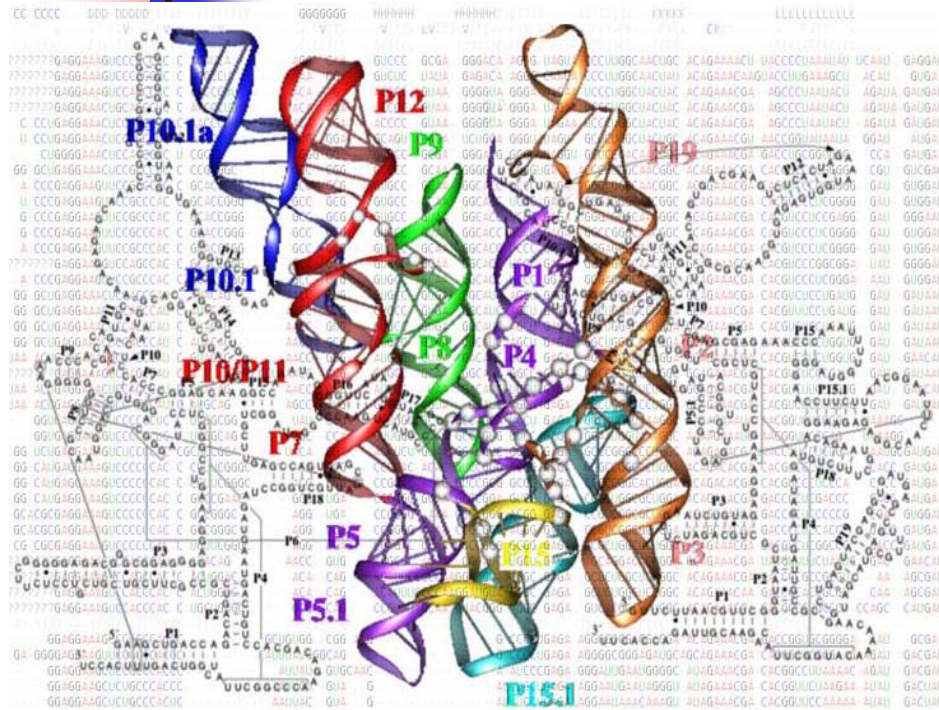
By A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

The “computable” numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means. Although the subject of this paper is ostensibly the computable *numbers*, it is almost equally easy to define and investigate computable functions of an integral variable or a real or computable variable, computable predicates, and so forth. The fundamental problems involved are, however, the same in each case, and I have chosen the computable numbers for explicit treatment as involving the least cumbersome technique. I hope shortly to give an account of the relations of the computable numbers, functions, and so forth to one another. This will include a development of the theory of functions of a real variable expressed in terms of com-

[Davis, M. : *The Undecidable*, Raven Press]

Why sequences in ML?(2)



- Many data for academic research is now open. In particular, many string data are provided in the area of **bio-informatics**.

5' C A C A U G U A C A A G A C U U 3'

Rfam: RNA families database of alignments and CMs

Home Keyword Search Sequence Search Browse Rfam Genomes ftp Help miRNA

Rfam Home Page

Rfam is a joint project involving researchers based at the [Wellcome Trust Sanger Institute](#), Cambridge, UK and [Janelia Farm](#), Ashburn, VA, USA. Rfam is a large collection of multiple sequence alignments and covariance models covering many common non-coding RNA families. For each family in Rfam you can:

- View and download multiple sequence alignments
- Read family annotation
- Examine species distribution of family members
- Follow links to otherdatabases

In conjunction with the [INFERnal](#) software suite, Rfam can be used to annotate sequences (including complete genomes) for homologues to known non-coding RNAs. Please read important information about [using Rfam for genome annotation](#). We provide pre-calculated lists of putative RNAs in over [200 complete genomes](#), and a [web search facility](#) for short sequences.

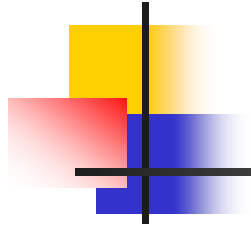
Rfam makes use of a large amount of available data, especially published multiple sequence alignments, and repackages these data in a single searchable and sustainable resource. We have made every effort to credit individual sources on family pages, and have compiled a list of links to these sources [here](#). If you find any of the data presented here useful, please also be sure to credit the primary source.

For more information on Rfam, and using this site, click [here](#).

Rfam Mirror Servers Worldwide FTP access to Rfam

[Sanger Institute \(UK\)](#) You can also download the Rfam database and for instance search it locally using the [INFERnal](#) covariance model software. [Hyperlink directly to the ftp site or View ftp site files](#)

[Janelia Farm \(USA\)](#)



FORMALIZATION OF THE PROBLEM



Formalization of the Task

Inputs : a (long) string S

a minimum number of
appearance(threshold) $\sigma \in \mathbf{N}$

Enumerate all patterns P satisfying $\text{supp}(P) \geq \sigma$

- Here a pattern P is defined as a **substring**.
- The support of P means how many times P occurs in S .



Substring and Subsequence

For a string $S = a_0a_1\dots a_n$

- A substring of S is $S[i, j] = a_i a_{i+1} a_{i+2} \dots a_j$
 - The terminology “**pattern** matching” means to find all substrings which is identical to a given string T .
- A subsequence of S is $a_{i_1} a_{i_2} a_{i_3} \dots a_{i_k}$ $0 \leq i_1 < i_2 < \dots < i_k \leq n$
 - just in the terminology “longest common subsequence”

例 For $S = \text{abcabcab}$ and $T = \text{ccaacaaba}$,

cabc is a substring and also a subsequence of S , while **cacb** is not a substring of S but a subsequence, and a common subsequence of S and T .



Suffix of a String

- A suffix of $S = a_0a_1\dots a_n$ is a subsequence $S[i, n]$ ($i = 1, 2, \dots, n-1$) ending with a_n .
 - We assume a_n is a special symbol \$

Example $S = \text{sakurasaku\$}$

$S[0,10] = \text{sakurasaku\$}$

$S[1,10] = \text{akurasaku\$}$

$S[2,10] = \text{kurasaku\$}$

$S[3,10] = \text{urasaku\$}$

$S[4,10] = \text{rasaku\$}$

$S[5,10] = \text{asaku\$}$

$S[6,10] = \text{asaku\$}$

$S[7,10] = \text{aku\$}$

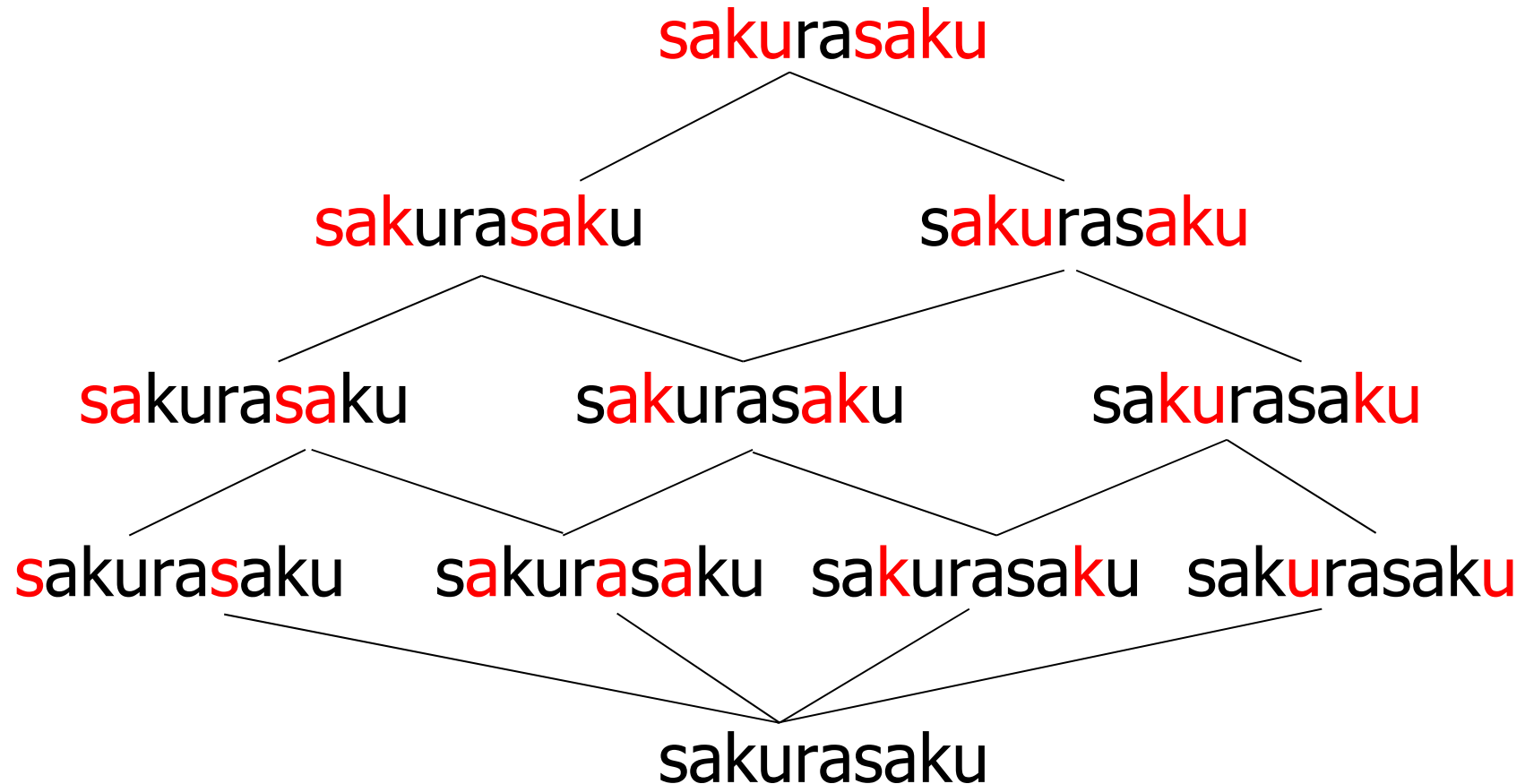
$S[8,10] = \text{ku\$}$

$S[9,10] = \text{u\$}$

Example (Tsuboi[2003])

$S = \text{sakurasaku}$

minimum support $\sigma = 2$





Tsuboi's Algorithm [Tsuboi 03]

Inputs :

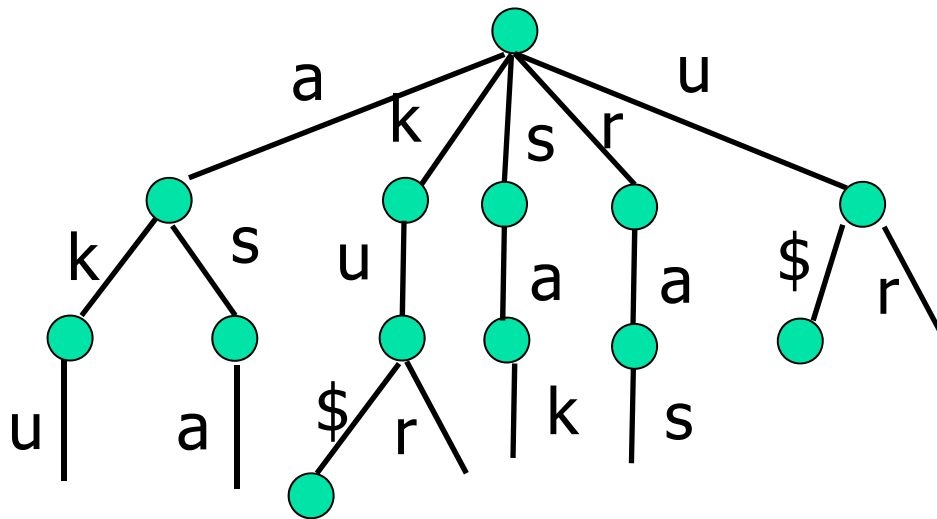
a **(long) string** S ending with a special symbol $\$$

a minimum number $\sigma \in \mathbf{N}$ of appearance (threshold)

1. Enumerate all suffixes of S
 2. Make three clusters of the suffixes.
 3. For each cluster obtained by Step 2, apply Step 2,
so we get a tree whose nodes has at most 3 children.
- /* This is hierarchical clustering. */

Prefix Tree (Trie) of all Suffixes

- Construct the prefix tree (trie) of all suffixes $S[i, n]$ ($i = 1, 2, \dots, n-1$).





A Step in Hierarchical Clustering (1)

- Just like the Quick Sort Algorithm, choose a pivot v and make three clusters with the head symbol $S[i]$ of each suffix.
 - If all suffixes have the same head symbol, try to use the second character, third character,...

Example $v = k$

$S[i] < v$

akurasaku\$

asaku\$

aku\$

$S[i] = v$

kurasaku\$

ku\$

$S[i] > v$

sakurasaku\$

urasaku\$

rasaku\$

sa\$

u\$



A Step in Hierarchical Clustering (2)

$v = k$

akurasaku\$
asaku\$
aku\$

kurasaki\$
ku\$

sakurasaku\$
urasaku\$
rasaku\$
saku\$
u\$

$v = s$

akurasaku\$
aku\$

asaku\$

$v = r$

kurasaki\$
ku\$

$v = s$

rasaku\$

sakurasaku\$
saku\$

urasaku\$
u\$



Pruning Clusters (1)

- $\text{supp}(S[i, j]) \geq \text{supp}(S[i-k, j]) \quad (k > 0)$
- We can delete all clusters which has suffixes less than σ .



Pruning Clusters (2)

$v = k$

akurasaku\$
asaku\$
aku\$

kur~~asaku~~\$
ku\$

sakurasaku\$
urasaku\$
rasaku\$
saku\$
u\$

$v = s$

akurasaku\$
aku\$

asaku\$

$v = r$

kur~~asaku~~\$
ku\$

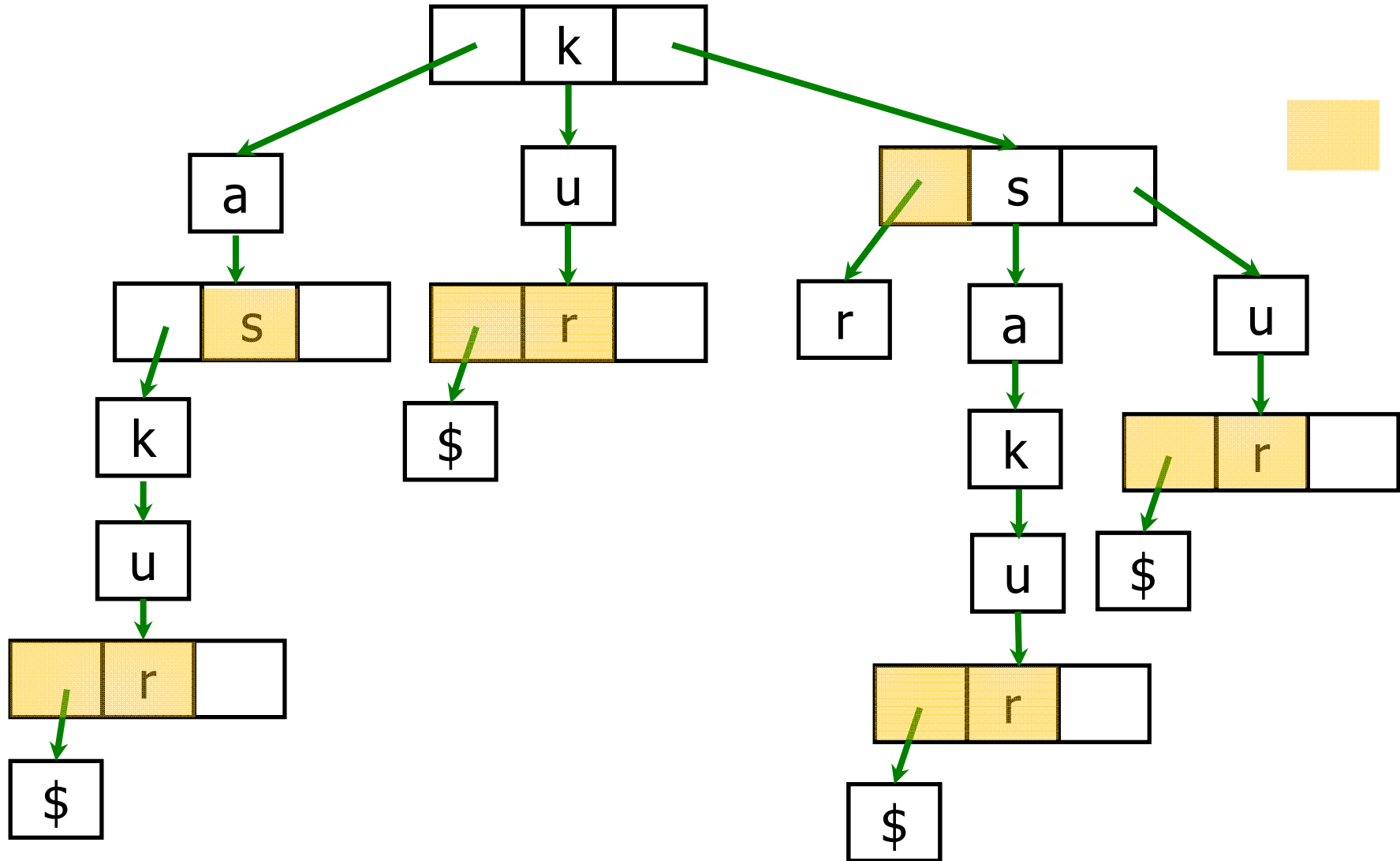
$v = s$

rasaku\$

sakurasaku\$
saku\$

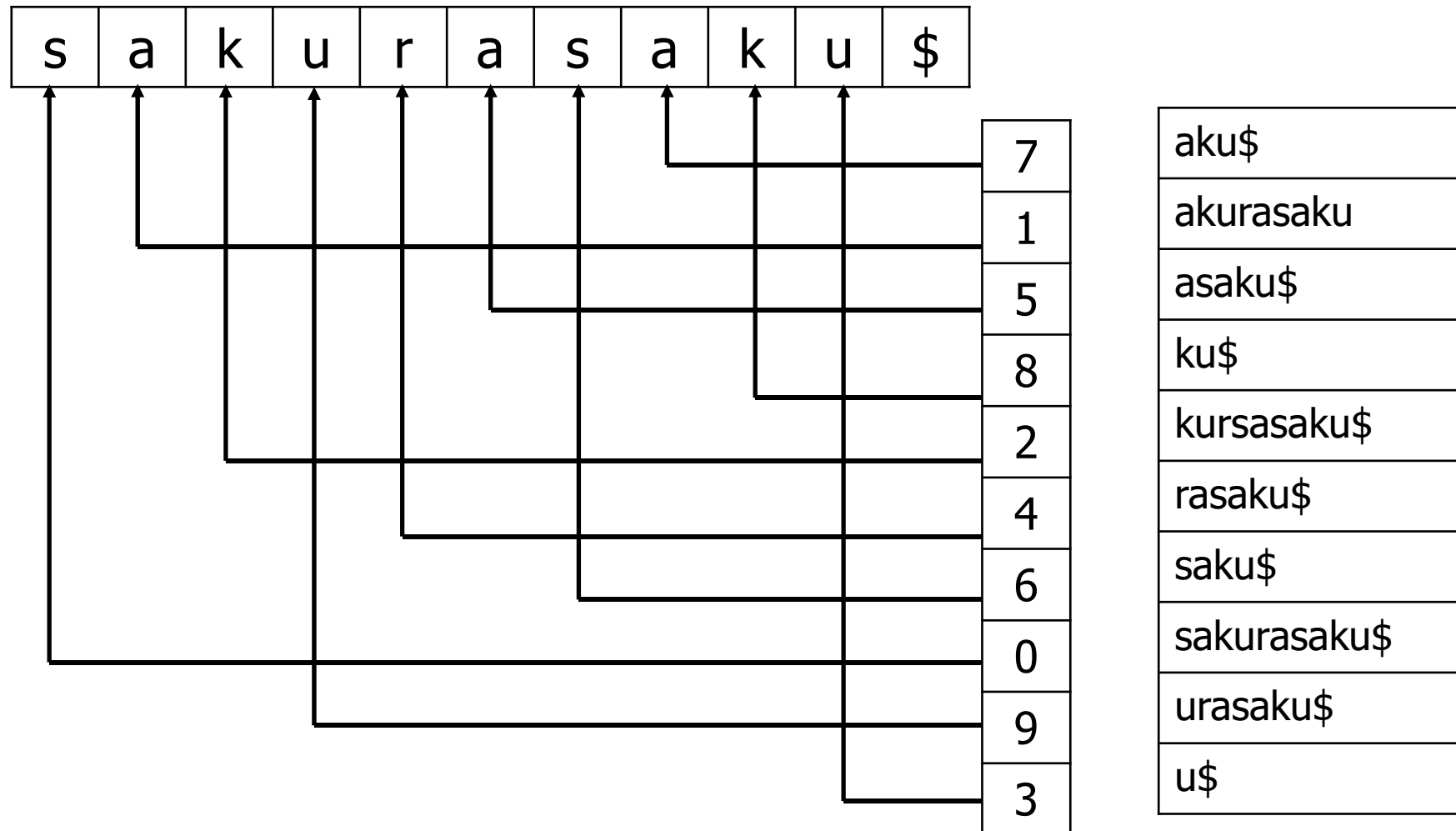
urasaku\$
u\$

Implementation with a Tree.



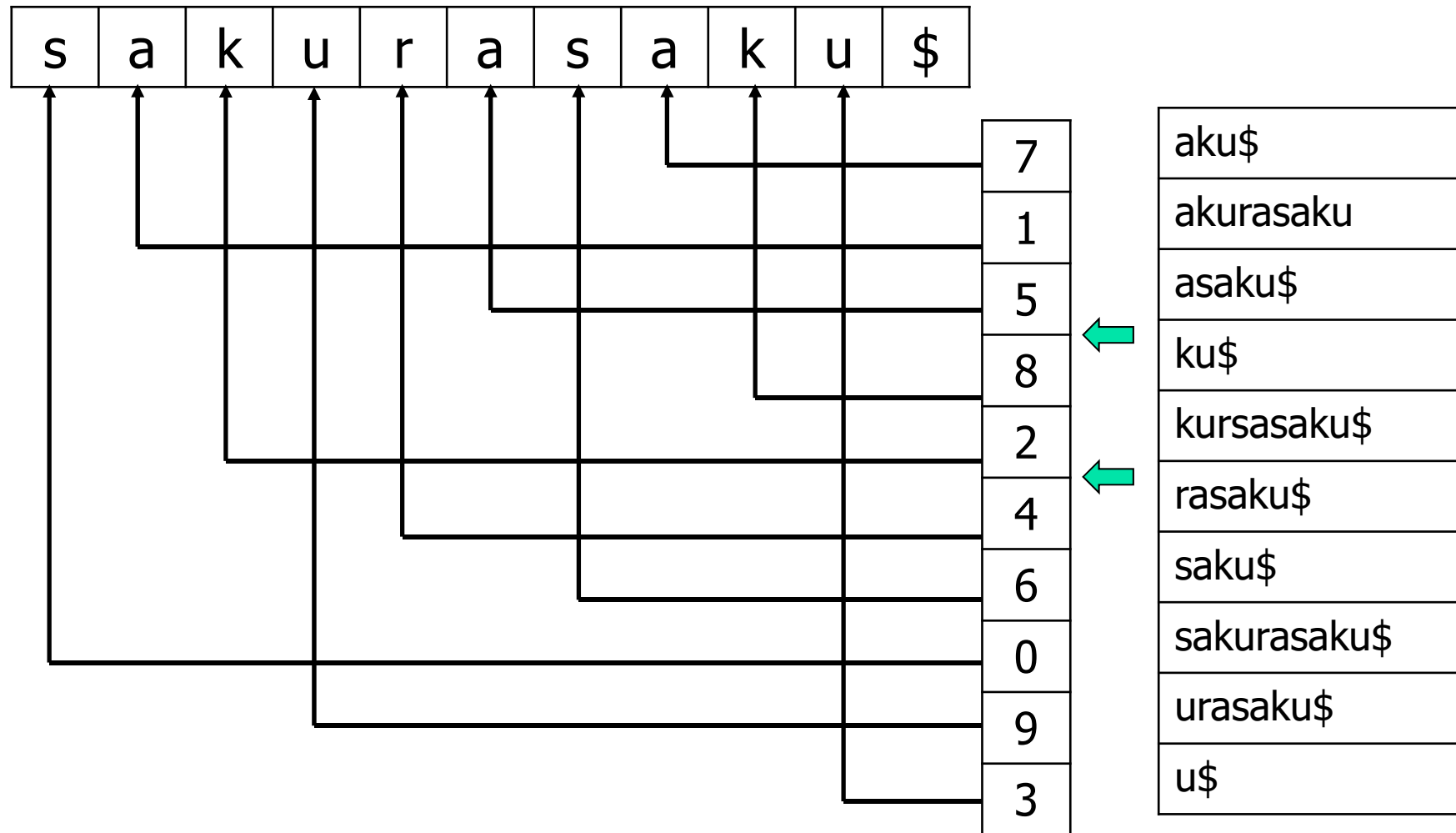
Implementation with an Array(1)

- Use the suffix array



Implementation with an Array(2)

- Divide the suffix array into three recursively.



Implementation with an Array(3)

- Divide the suffix array into three recursively.

