Computational Learning Theory
Learning Automata with Queries

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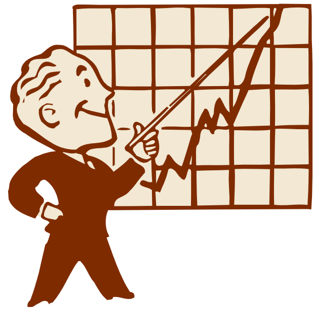
Introduction

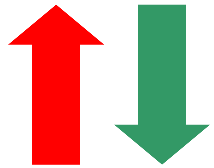
- Every FA *M* is identified with the examples in the characteristic set for *M*.
  - The characteristic set is generated with a minimal test set and an observation table.
  - The observation table is constructed with a prefix closed set.
  - A prefix closed set is "dense" in the sense that no prefix of any element in the set is missing.
- When we have some missing examples, how should we do?

  One solution is to estimate the missing examples with some properties of FA.
  Another solution is to revise the learning machine so that it can request the missing examples.

Learning with Queries

- One of the framework of “identification in the limit” is modeling a passive learner.
- We consider active algorithms for learning, which request information necessary to conjecture targets.
- By allowing machines to use queries we could make them more active.
- We assume a teacher or an oracle.

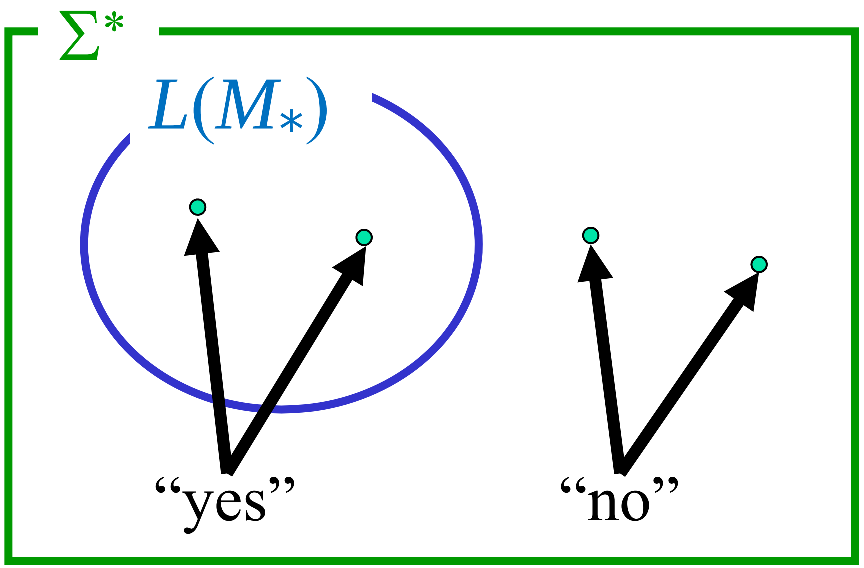




Learning FAs[Angluin87]

- We introduce an algorithm that learns any finite state automaton M_* on Σ using the two types of queries;

  1. A membership query MQ(w) for a string in
     $w \in \Sigma^*$ asking "Does w belong to $L(M)$?"
     The answer is "*yes*" or "*no*".



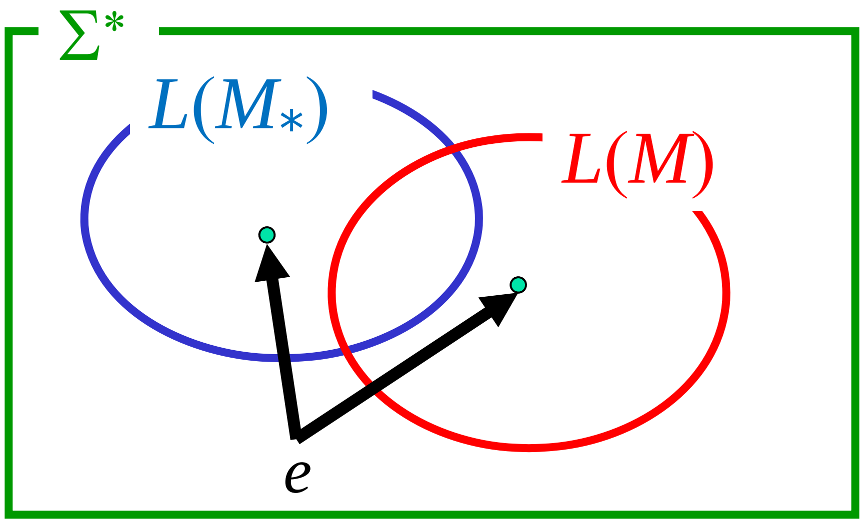
Learning FAs[Angluin87]

2. A equivalence query EQ(M) for the current my conjecture M asking "$L(M)=L(M_*)$?"

   The answer is "*yes*" or

   a counter example e such that

   $$e \in (L(M) - L(M_*)) \cup (L(M_*) - L(M))$$

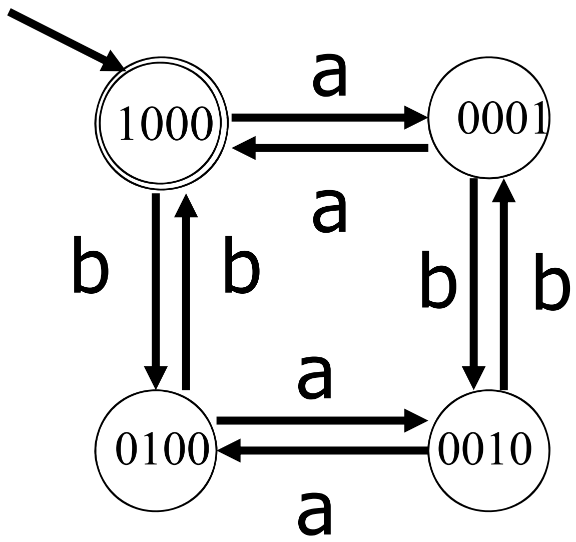


Observation table

- An observation table (S, E, T) :
  S : a prefix closed set $S \subset \Sigma^*$
  E : a suffix closed set $E \subset \Sigma^*$
  $T : (S \cup S\,\Sigma)E \rightarrow \{0, 1\}$
  - A set of strings S is prefix closed (suffix closed) if and only if every prefix (resp. suffix) of every member of S is also a member of S.
  - $S\,\Sigma = \{ sa \mid s \in S \text{ and } a \in \Sigma \}$
  - The element of the position (s, w) shows that the automaton M for the current conjecture accepts sw.

Observation table(cont.)

- Intuitively, each row of the *S* part represents a state in an automaton *M* and each row of the *S* Σ part represents a transition.

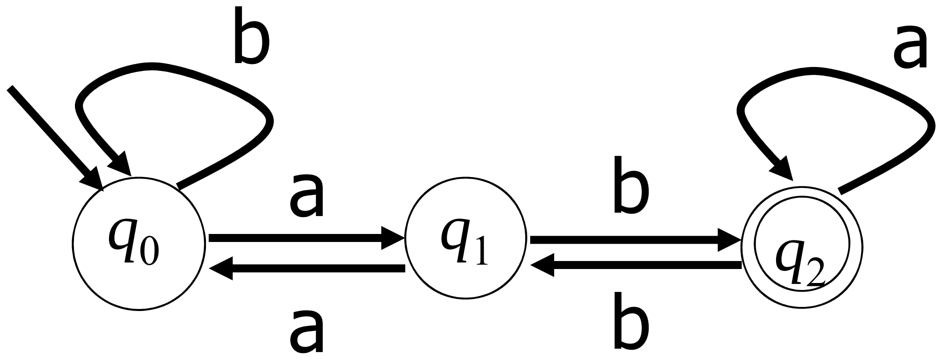


E

	ε	b	ab	bab
ε	1	0	0	0
a	0	0	0	1
b	0	1	0	0
ab	0	0	1	0
aa	1	0	0	0
ba	0	0	1	0
bb	1	0	0	0
aba	0	1	0	0
abb	0	0	0	1

Example 1 (1)

- The target finite state automaton M_* is:



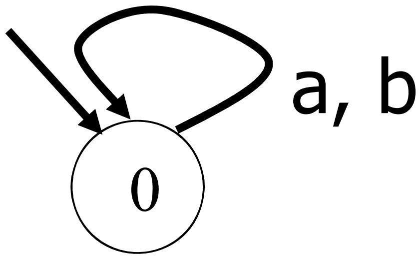
- This automaton is not known by the learning algorithm.

Example 1 (2)

- Make the initial observation table T_1 with MQ(a) and MQ(b).
  - The S part consists of row(ε) and
  - the $S\Sigma$ part consists of row(a) and row(b).

		ε
S	ε	0
$S\Sigma$	a	0
	b	0

- T_1 is consistent and closed, and therefore represents the finite state automaton:



- This accepts no string and is not equivalent to the target automaton M_* and the teacher gives a counter example.

Example 1 (3)

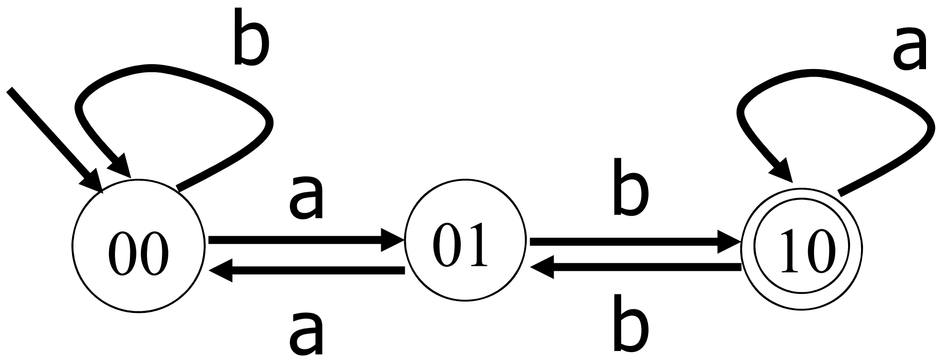
- Assume that give ab is given by the teacher as a counter example.
- Then add ab and its prefixes to *S*.
  Also make the columns for aa, aba, abb.
- Extend the table with MQ(aa), MQ(aba), and MQ(abb).

	ε
ε	0
a	0
ab	1
b	0
aa	0
aba	1
abb	0

- The table is closed but not consistent because row(ε)=row(a) but row(εb)≠row(ab).

Example 1 (4)

- Add b to E and extend the table with MQ(bb), MQ(aab), MQ(abab), and MQ(abbb).
- The table is consistent and closed, and represents the automaton:



	ε	b
ε	0	0
a	0	1
ab	1	0
b	0	0
aa	0	0
aba	1	0
abb	0	1

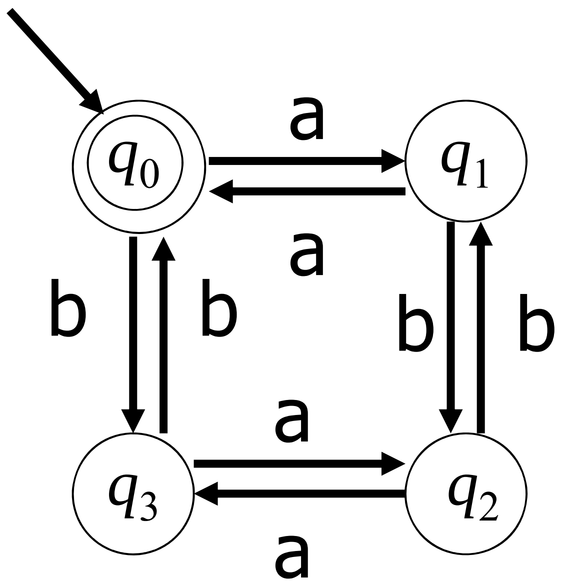
Consistent tables and Closed tables

- An observation table (S, E, T) is consistent if and only if for every pair $w, v \in S$ such that $\text{row}(w)=\text{row}(v)$, $\text{row}(wc)=\text{row}(vc)$ for any $c \in \Sigma$.
  - Intuitively, in a consistent table, every row in the S part can be regarded as one state of an automaton.
- An observation table (S, E, T) is closed if for every $w \in S\Sigma$ there exists $v \in S$ such that $\text{row}(w)=\text{row}(v)$.
  - Intuitively, in a closed table, every row in the $S\Sigma$ part can be interpreted as a transition of an automaton.
- From a closed and consistent observation table (S, E, T), we define a finite state automaton $M(S, E, T)$ as follows:

$$Q = \{\text{row}(w) : w \in S\},\ q_0 = \text{row}(\varepsilon),$$
$$F = \{\text{row}(w) : w \in S \text{ and } T(w) = 1\},$$
$$\delta(\text{row}(w),c) = \text{row}(wc)$$

Example 2 (1)

- The target finite state automaton M_* is:



Example 2 (2)

- Make the initial observation table T_1 with MQ(a) and MQ(b).
  - The S part consists of row(ε) and the $S\Sigma$ part consists of row(a) and row(b).

		ε
S	ε	1
$S\Sigma$	a	0
	b	0

Example 2 (3)

- T_1 is consistent but not closed because row(a) $\neq$ row(w) for all $w \in S$.
  Add a to S.

	ε
ε	1
a	0
b	0

S = {ε}; $S\Sigma$ = {a, b}

⇩

	ε
ε	1
a	0
b	0

S = {ε, a}

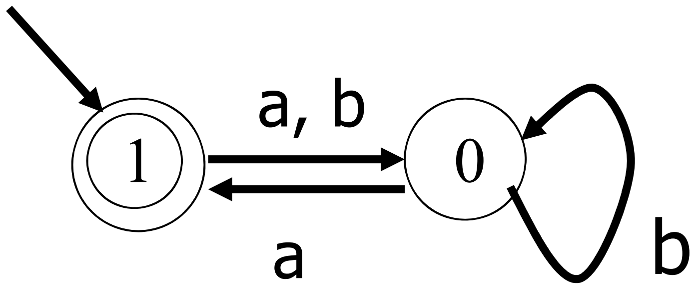
- But then T_1 misses row(aa) and row(ab) for $S\Sigma$.
  - To add some rows to T for keeping the definition of an observation table is called to make a closure of T.

Example 2 (4)

- Make the closure T_2 of T_1 with MQ(aa) and MQ(ab).
- T_2 is closed and consistent and represents the automaton below.
- make EQ(M(S, E, T))

		ε
S	ε	1
	a	0
	b	0

		ε
	ε	1
	a	0
$S\,\Sigma$	b	0
	aa	1
	ab	0



Example 2 (5)

- Because M(S, E, T) is not equivalent to the target, a counter example, say bb, is given by the teacher.

  Add bb to S and make T_3

  with MQ(bb), MQ(ba), MQ(bab), and MQ(bbb).

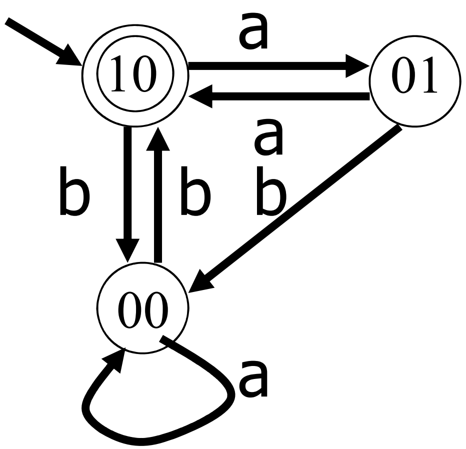
- T_3 is closed but not consistent because row(a)=row(b) but row(aa)≠row(ba).

	ε
ε	1
a	0
b	0
bb	1
aa	1
ab	0
ba	0
bba	0
bbb	0

Example 2 (6)

- Add a to E and make T_4 with MQ(aaa), MQ(aab), MQ(baa), MQ(bbaa), MQ(bbba)
- T_4 is closed and consistent because row(ε)=row(bb), row(a)=row(bba) and row(b)=row(bbb).

  make EQ(M(S, E, T))



	ε	a
ε	1	0
a	0	1
b	0	0
bb	1	0
aa	1	0
ab	0	0
ba	0	0
bba	0	1
bbb	0	0

Example 2 (7)

- Because M(S, E, T) is not equivalent to the target, a counter example, say abb, is given by the teacher.

  Add ab, abb to S and make T_5

  with MQ(abb), MQ(abba),

  MQ(aba), MQ(abaa),

  MQ(abba), MQ(abbaa),

  MQ(abbb), MQ(abbba).

- T_5 is not consistent because

  row(b)=row(ab) and row(bb) ≠ row(abb).

	ε	a
ε	1	0
a	0	1
b	0	0
bb	1	0
ab	0	0
abb	0	1
aa	1	0
ba	0	0
bba	0	1
bbb	0	0
aba	0	0
abba	1	0
abbb	0	0

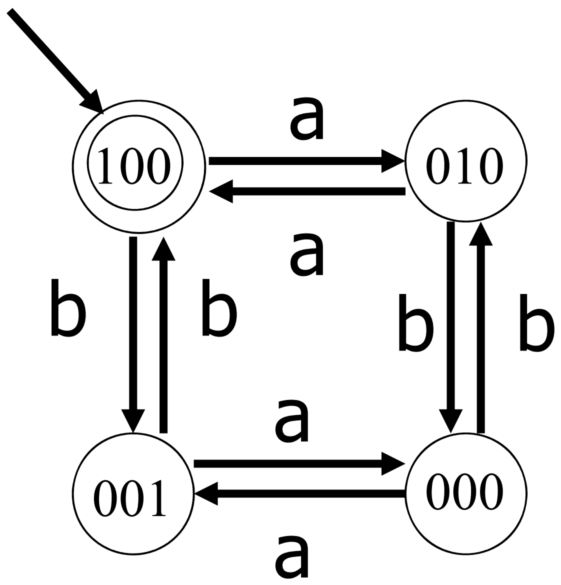
Example 2 (8)

- Add b to E and make T_6
  with MQ(aab), MQ(bab),
  MQ(bbab), MQ(bbbb),
  MQ(abab), MQ(abbab), MQ(abbbb).
  T_6 is closed and consistent because
  row(ε)=row(bb),
  row(a)=row(bba), row(b)=row(bbb),
  row(a)=row(abb),
  row(aa)=row(abba), row(ab)=row(abbb)

	ε	a	b
ε	1	0	0
a	0	1	0
b	0	0	1
bb	1	0	0
ab	0	0	0
abb	0	1	0
aa	1	0	0
ba	0	0	0
bba	0	1	0
bbb	0	0	1
aba	0	0	1
abba	1	0	0
abbb	0	0	0

Example 2 (9)

- T_6 represents the automaton below.



	ε	a	b
ε	1	0	0
a	0	1	0
b	0	0	1
bb	1	0	0
ab	0	0	0
abb	0	1	0
aa	1	0	0
ba	0	0	0
bba	0	1	0
bbb	0	0	1
aba	0	0	1
abba	1	0	0
abbb	0	0	0

Algorithm

S:={ε}, E:={ε}

Ask membership queries for ε and every $c \in \Sigma$

Construct the initial observation table (S, E, T)

Repeat

  While (S, E, T) is not closed or not consistent

    If (S, E, T) is not consistent

      extend-for-consistency(S, E, T)

    If (S, E, T) is not closed

      make-closure(S, E, T)

  M:=M(S, E, T) and make EQ(M)

  If the teacher replies with a counter example e,

    add e and all prefixes to S

    for each prefix p of e (including e) and each $u \in E$

      MQ(pu)

    extend T

  Else break the loop and exit with returning M(S, E, T)

Extend a Table for Consistency

extend-for-consistency(S, E, T)

/* (S, E, T) is not consistent */

Find w, $v \in S$, $c \in \Sigma$, and $e \in E$ such that

row(w)=row(v) but $T(wce) \neq T(vce)$

add ce to E

for each $u \in (S \cup S\,\Sigma)$

ask MQ(uce)

extend T

Make a Closure of a Table

make-closure(S, E, T)

/* (S, E, T) is not closed */

Find $w \in S$ and $c \in \Sigma$ such that

row(wc) $\neq$ row(v) for all $v \in S$

add wc to S

for each $u \in E$

MQ(wcu)

extend T

Notes

- Thanks to the equivalence queries, the algorithm L^* can know whether or not the current conjecture is correct.
  - Note: In the framework of identification in the limit, learning an algorithm cannot know whether or not the current conjecture is correct.
- The size of the observation table is bounded by

  $$(|\Sigma|+1)\ (n+(m-1))n\ (m+2n-1) = O(m^2n^2+mn^3)$$

  where n is the number of the minimal FA equivalent to $L(M_*)$ and m is the maximum length of counter examples provided by the teacher.

Example (5)'

- Because M(S, E, T) is not equivalent to the target, a counter example, say abab, is given by the teacher.

  Add abab to S and make T_3'

  with MQ(abab), MQ(aba), …

	ε
ε	1
a	0
b	0
ab	0
aba	0
abab	1
aa	1
ba	0
bb	0
	0
abaa	0
ababa	0
ababa	0

The Myhill-Nerode Theorem

Theorem The following three statements are equivalent:
(1) The language L is accepted by some finite automaton.
(2) L is the union of some equivalence classes of a right invariant equivalence relation of finite index.
(3) Let equivalence relation R_L be defined by: $x\ R_L\ y$ if and only if for all $z \in \Sigma^*$ xz is in L iff yz is in L. Then R_L is finite index.

- An equivalence relation R is right invariant iff $x\ R\ y$ implies $xz\ R\ yz$ for all $z \in \Sigma^*$.
- The index of equivalence relation R is the number of equivalence classes.

Reference

- D. Angluin:

  Learning Regular Languages from Queries andCounter-Examples, *Information and Computation* 75(2), 87-106, 1987