Computational Learning Theory
Extending Patterns with Deductive Inference

Akihiro Yamamoto 山本 章博
http://www.iip.ist.i.kyoto-u.ac.jp/member/akihiro/
akihiro@i.kyoto-u.ac.jp

Contents

Examples

Example 2

$C_2 = \{$ba, bababa, babababa, bababababa$\}$

$D_2 = \{$a, b, bbbb, abb, baaaaba, babbb$\}$

- It might hold that *every string in* C_2 *is made of some repetition of* ba.

Example 3

$C_3 = \{$aaabbb, ab, aaaabbbb, aaaaabbbbb, aabb$\}$

$D_3 = \{$a, b, bbbb, abb, baaaaba, babbb$\}$

- *Every string in* C_3 *consists of two strings: The first string consists only of* a*'s, and the second consists of the same number of* b*'s.*

Examples

Example 5

Let Σ = {Kei, play, tennis, Naomi, Alexa, Pepper}

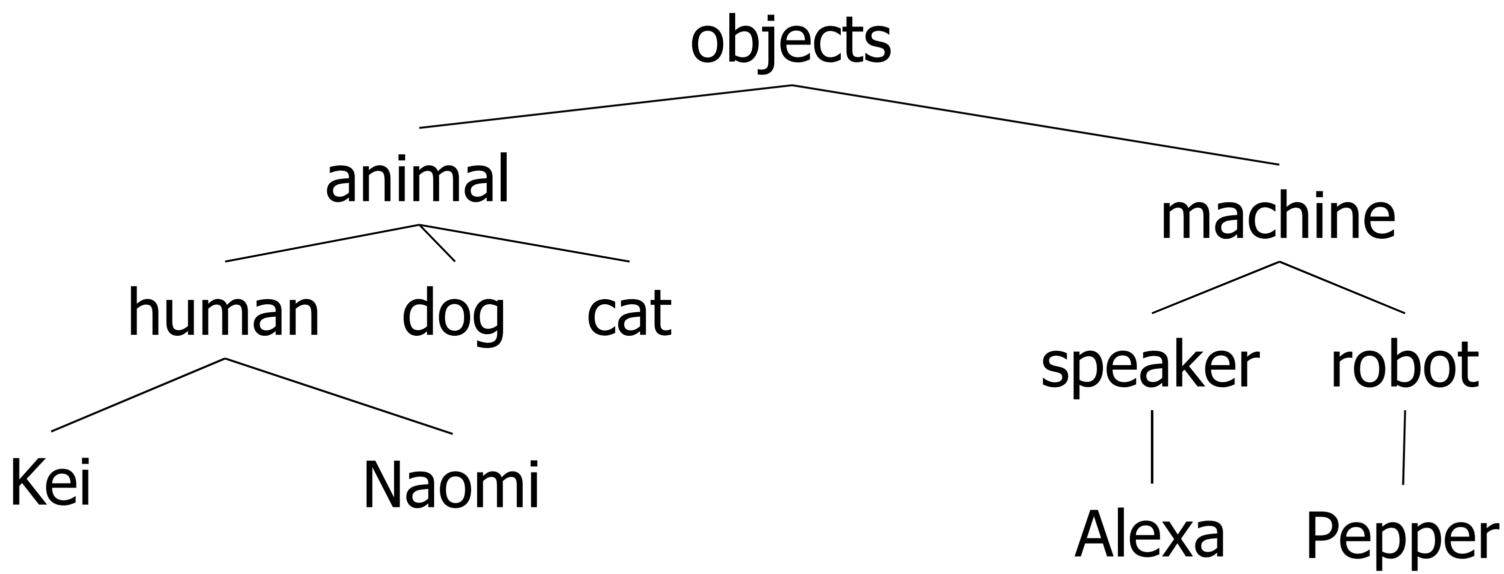
C_5 = {Kpt, Npt}

D_5 = {Apt, Ppt}

- Assume that we know that both Socrates and Tom is human, and both Alexa and Pepper are machines, we might conjecture that "All humans are mortal."

Assumption

- Each symbol has types.
  - Each symbol belongs to some classes.
  - The class may not be unique.
- All types constitute a DAG i.e. a Hasse diagram.
  - DAG : Directed Acyclic graph.
  - Some partial ordering relation is defined for the set of types.



LCG

- Let a partial order ≥ is defined for elements in *S*.
  An element $g \in S$ is a common generalization of
  $a_1, a_2, \ldots,$ and a_n if $g \geq a_1, g \geq a_2, \ldots,$ and $g \geq a_n$.
- A common generalization *g* of $a_1, a_2, \ldots,$ and a_n is the least common generalization if $g' \geq g$ for g' of any their common generalization.
- For sequences of a same length, a partial ordering ≥ is defined as :

  $a_1 a_2 \ldots a_n \geq b_1 b_2 \ldots b_n$ iff

  $a_1 \geq b_1, a_2 \geq b_2, \ldots,$ and $a_n \geq b_n$

  and so as common generalization.

Ordering of patterns

- A partial order of patterns are defined as:

  $\pi \geq \tau$ iff $\tau = \pi\theta$ for some substitution θ.

Example

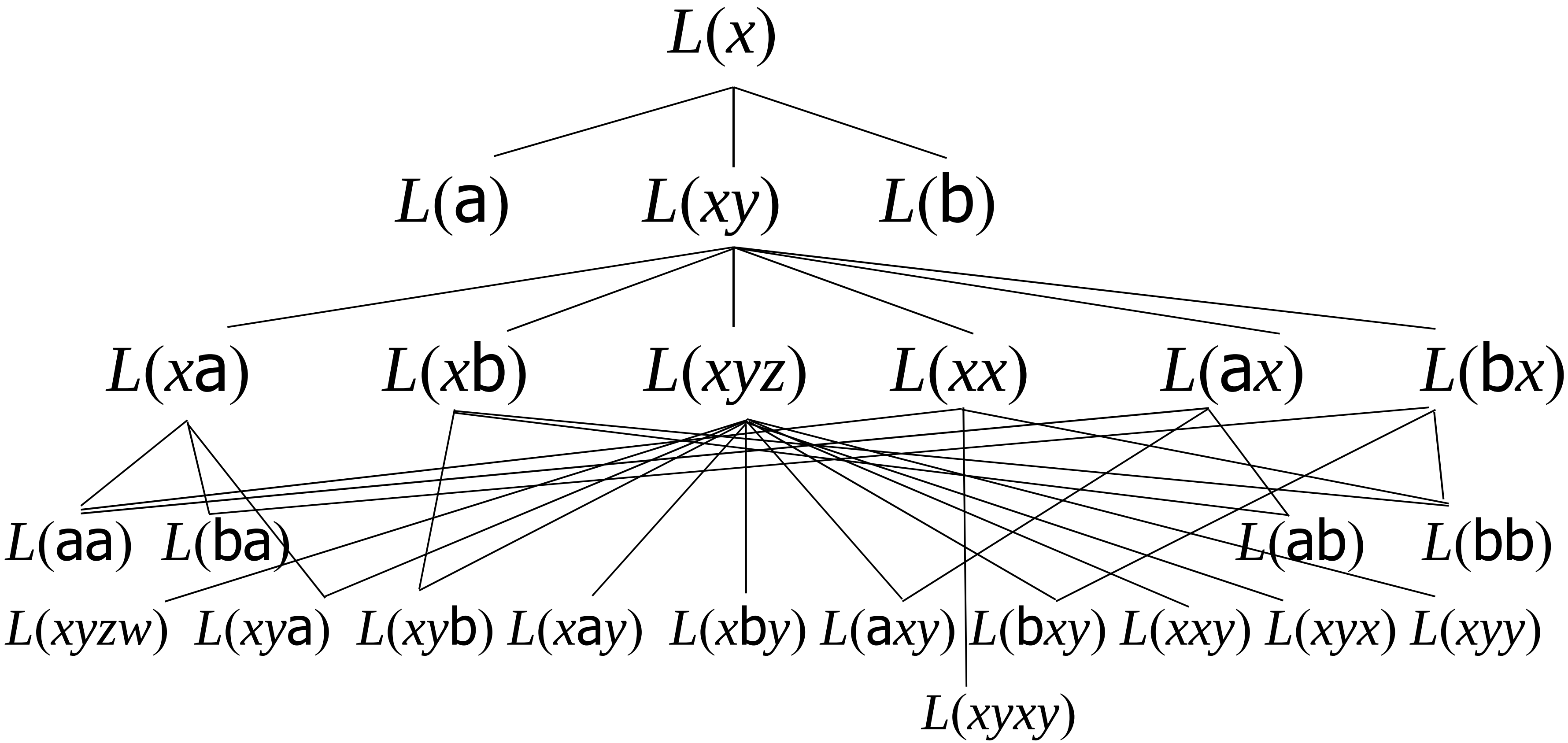
bxaxb $\geq$ bbbaabbab because

bxaxbθ_1 = bbbaabbab for θ_1 ={ (x, bba), (y, ba) }

a$x$bb$y$aθ_2 = ab$y$abba$y$ba because

a$x$bb$y$aθ_2 = ab$y$abba$y$ba for θ_2 ={ (x, bya), (y, ayb) }

Hasse Diagram (General Version)



Example

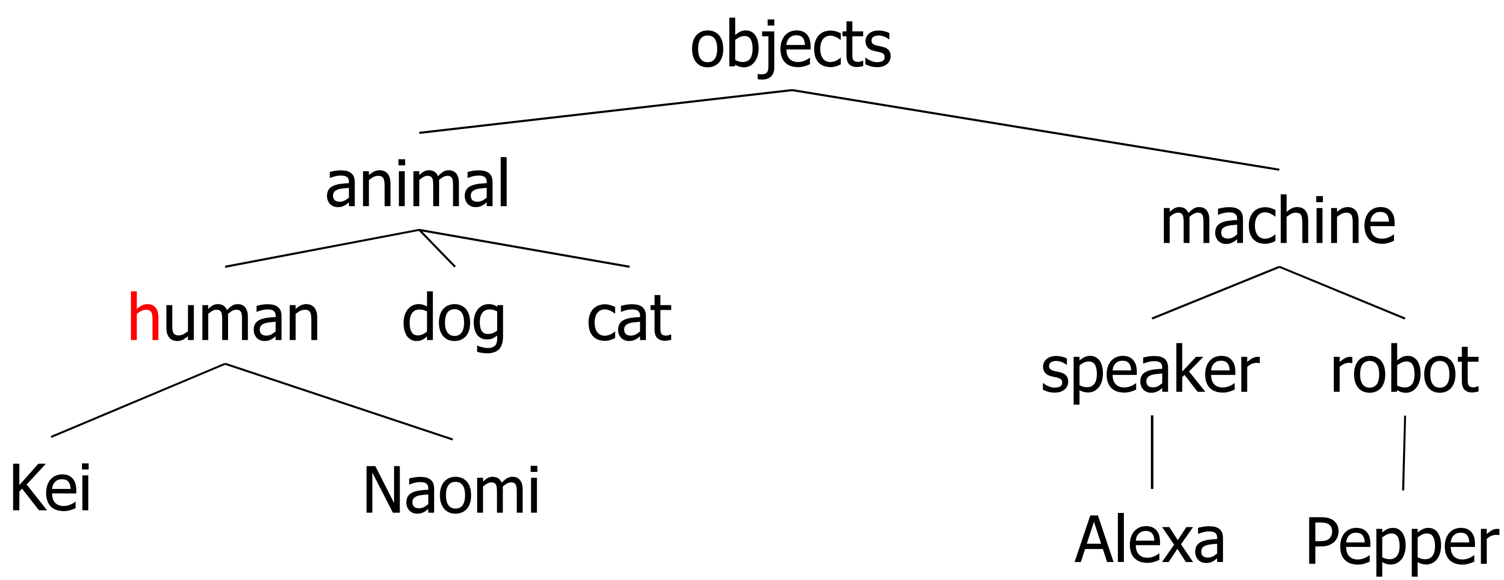
Example 5

Let Σ = {Kei, play, tennis, Naomi, Alexa, Pepper}

C_5 = {Kpt, Npt}

D_5 = {Apt, Ppt}

The least common generalization of the two sequences in C_5 is hpt



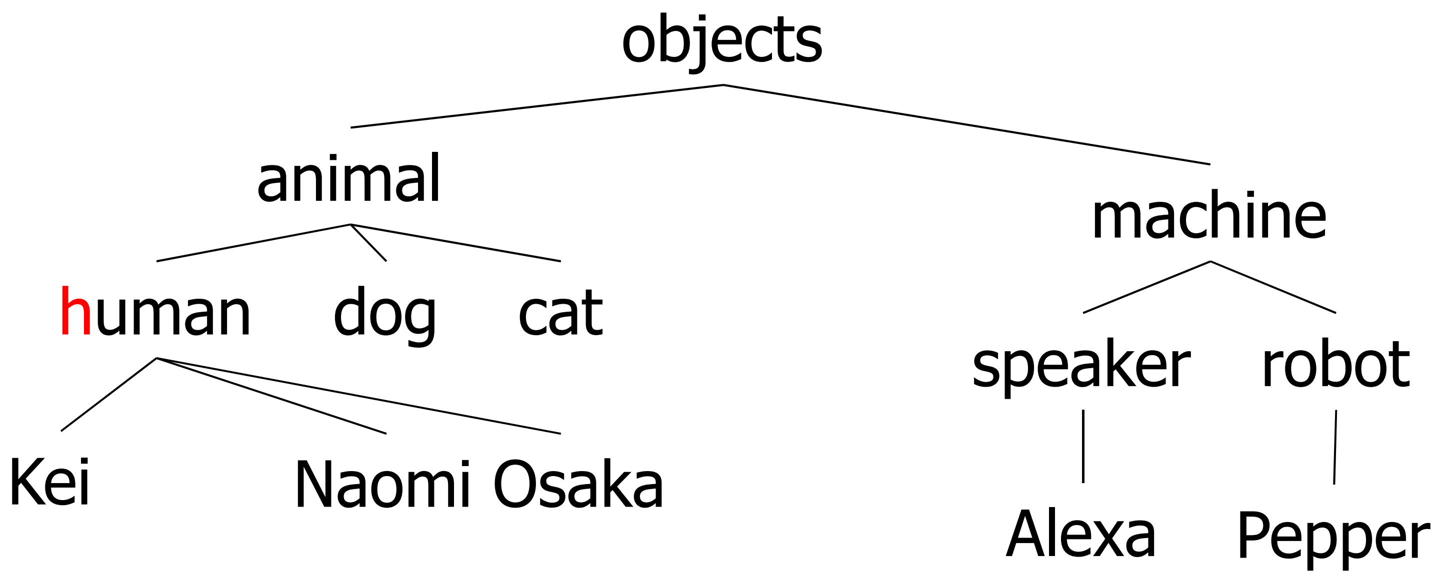
Different Lengths

Example 6

Let Σ = {Kei, play, tennis, Osaka, Naomi, Alexa, Pepper}

C_6 = {Kpt, NOpt}

D_6 = {Apt, Ppt}



Anti-Unifcation of Strings

- For a set C of stings of same length

$$
\begin{aligned}
s_1 &= c_{11}\, c_{12} \ldots c_{1i} \ldots c_{1k} \\
s_2 &= c_{21}\, c_{22} \ldots c_{2i} \ldots c_{2k} \\
&\ldots \\
s_n &= c_{n1}\, c_{n2} \ldots c_{nj} \ldots c_{nk}
\end{aligned}
$$

the anti-unification of C is a pattern

$$\pi = \gamma(c_{11}c_{21}\ldots c_{n1})\gamma(c_{12}c_{22}\ldots c_{n2}) \ldots \gamma(c_{1k}c_{2k}\ldots c_{nk})$$

where

$$\gamma(c_1c_2\ldots c_n) = \begin{cases} c & \text{if } c_1 = c_2 = \ldots = c_n = c \\ x_{\iota(c1c2\ldots cn)} & \text{otherwise.} \end{cases}$$

and $\iota(c_1c_2\ldots c_n)$ is the "index" of $c_1c_2\ldots c_n$.

Analysis of Patterns (2)

Example π = axxbb$y$aa

L(axxbb$y$aa)

={aaabbaaa, aaabbbaa, abbbbaaa, abbbbbaa, aaaaabbaaa, aaaaabbbaa, aababbbaaa, aababbbbaa,…, aabaaabaabbbbbababaa,…}

- Using examples as long as π :

aaabbaaa, aaabbbaa, abbbbaaa, abbbbbaa

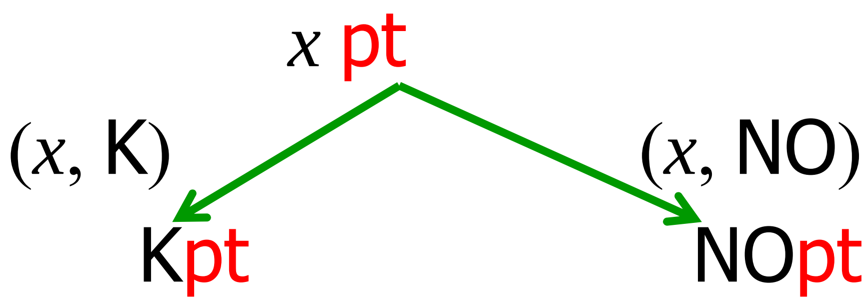
θ_1={(x,a), (y,a)} θ_2={(x,a), (y,b)} θ_3={(x,b), (y,a)} θ_4={(x,b), (y,b)}

We can know that the 2nd, 3rd, and 6th positions must be variables.

The variable at the 6th position is different from those at the 2nd and 3rd.

Using Typed Variables

- When introducing a variable, check all the strings substituted to it have a same type.



Do K and NO have a same type?

- To answer the question, we have to treat two structure:

  semantical structure and syntactic structure

How to treat the two structure

- Introducing a new class of symbols called predicate symbols.
- Applying logical inference to check the structure.
  - The idea came from Mathematical Logic.

animal(x) if human(x)

objects(x) if animal(x)

human(xy) if human(x) and human(y)

Elementary Formal Systems

Mathematical Logic

- Mathematical logic is a subfield of mathematics exploring the applications of formal logic to mathematics.[Wikipedia]

All humans are mortal.

Socrates is human.

↓ Inference

Socrates is mortal.

$\text{mortal}(x) \leftarrow \text{human}(x)$

human(Socrates)

↓ Operation Algorithm

mortal(Socrates)

EDIA
yclopedia

ntent
ts
cle
ikipedia
hop

pedia
y portal
anges
ge

here
anges
e
ges
t link
mation

age

ook
as PDF
ersion

Raymond Smullyan

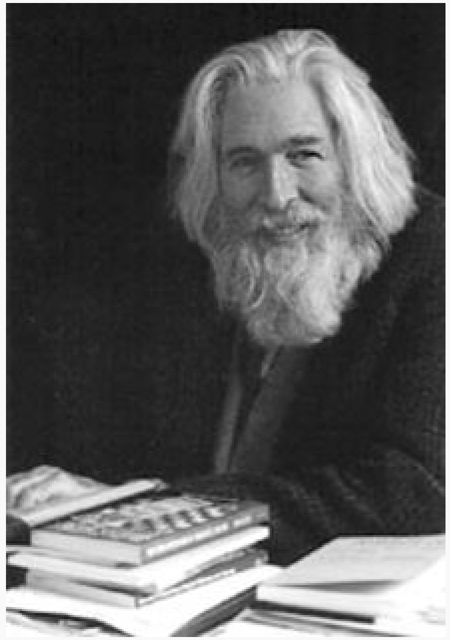
From Wikipedia, the free encyclopedia

This article includes a list of references, but **its sources remain unclear because it has insufficient inline citations**. Please help to improve this article by introducing more precise citations. *(February 2008)*

Raymond Merrill Smullyan (born May 25, 1919)[1] is an American mathematician, concert pianist, logician, Taoist philosopher, and magician.

Born in Far Rockaway, New York, his first career was stage magic. He then earned a BSc from the University of Chicago in 1955 and his Ph.D. from Princeton University in 1959. He is one of many logicians to have studied under Alonzo Church.[1]

Raymond Merrill Smullyan



Born	May 25, 1919 (age 95) Far Rockaway, New York
Occupation	mathematician, logician, philosopher, pianist and magician
Nationality	American

Contents [hide]

Life [edit]

Raymond Smullyan

- (1978) What Is the Name of This Book?
- (1980) This Book Needs No Title
- (1979) The Chess Mysteries of Sherlock Holmes
- (1982) The Lady or the Tiger?
- (1982) Alice in Puzzle-Land

- (2015) The Magic Garden of George B and Other Logic Puzzles

Predicate Symbols

- I
  - Every predicate symbol is interpreted as a language (a set of strings) or a set of tuples (s_1, s_2,…,s_n) of strings.
  - In this course we use symbols p, q, r,… which are respectively interpreted as sets P, Q, R,….
- An atomic formula is a formula of the form

$$p(\pi_1, \pi_2, \ldots, \pi_n)$$

where $\pi_1, \pi_2, \ldots, \pi_n$ are patterns. If n=1 and $\pi_1 = s$ is ground, p(s) is interpreted as $s \in$ P.

Example

Some examples of atomic formulae are p(axb), q(ax, by), q(x, bxb), p(aabb), q(aa, bb). The last two formulae are ground.

Definite Clause (Rules) and EFS

- An definite clause is a formula of the form

$$p(\pi_1,\ldots,\pi_n) \leftarrow q_1(\tau_{11},\ldots), q_2(\tau_{21},\ldots),\ldots, q_k(\tau_{k1},\ldots)$$

where $\pi_1, \pi_2,\ldots, \tau_{11},\ldots, \tau_{k1},\ldots$ are patterns. The definite clause is interpreted as

"for any substitution θ, if $(\tau_{11}\theta,\ldots) \in Q_1, (\tau_{21}\theta,\ldots) \in Q_2,\ldots,$ $q_k(\tau_{k1}\theta,\ldots) \in Q_k$ then $(\pi_1\theta, \pi_2\theta,\ldots, \pi_n\theta) \in P$"

  - A clause $p(\pi_1,\ldots,\pi_n) \leftarrow$ which has no conditions is sometimes called a unit clause.

- A finite set of definite clause is called an elementary formal system (EFS). [Smullyan 61]

Examples

Some examples of definite clauses are

$$p(ax) \leftarrow r(x)$$

$$r(b) \leftarrow$$

$$p(axby) \leftarrow r(x), r(y)$$

$$q(ax, by) \leftarrow q(x, y)$$

$$\dots$$

$$animal(x) \leftarrow human(x)$$

$$objects(x) \leftarrow animal(x)$$

$$human(xy) \leftarrow human(x),\ human(y)$$

$$human(K) \leftarrow$$

$$human(N) \leftarrow$$

Inference Rules for Definite Clauses

- A goal clause is a sequence (conjunction) of atomic formulae $p_1(\tau_{11},\ldots),\ldots, p_k(\tau_{k1},\ldots)$
- We use the following two rules

Instantiation

$$\frac{p(\pi_1,\ldots) \leftarrow q_1(\tau_{11},\ldots),q_2(\tau_{21},\ldots),\ldots, q_k(\tau_{k1},\ldots)}{(p(\pi_1,\ldots) \leftarrow q_1(\tau_{11},\ldots),q_2(\tau_{21},\ldots),\ldots, q_k(\tau_{k1},\ldots))\theta}$$

Modus Pones

$$\frac{p(\pi_1,\ldots) \leftarrow p_1(\pi_{11},\ldots),\ldots, p_k(\pi_{k1},\ldots) \quad p_1(\pi_1,\ldots) \leftarrow q_1(\tau_{11},\ldots),\ldots}{p(\pi_1,\ldots) \leftarrow q_1(\tau_{11},\ldots),\ldots, p_2(\pi_{21},\ldots),\ldots, p_k(\pi_{k1},\ldots)}$$

- A proof is a continuous application of the inference rules.

Example of Proof

$\mathrm{a}(x) \leftarrow \mathrm{h}(x)$

$\mathrm{o}(x) \leftarrow \mathrm{a}(x)$

$\mathrm{h}(xy) \leftarrow \mathrm{h}(x),\ \mathrm{h}(y)$

$\mathrm{h(K)} \leftarrow$

$\mathrm{h(N)} \leftarrow$

$\mathrm{h(O)} \leftarrow$

$$\dfrac{\dfrac{\dfrac{\mathrm{h}(xy) \leftarrow \mathrm{h}(x),\ \mathrm{h}(y)}{\mathrm{h(NO)} \leftarrow \mathrm{h(O)},\ \mathrm{h(N)}} \qquad \mathrm{h(O)} \leftarrow}{\mathrm{h(NO)} \leftarrow \mathrm{h(N)}} \qquad \mathrm{h(N)} \leftarrow}{\mathrm{h(NO)} \leftarrow}$$

Example of Proof

$$\mathrm{a}(x) \leftarrow \mathrm{h}(x)$$

$$\mathrm{o}(x) \leftarrow \mathrm{a}(x)$$

$$\mathrm{h}(xy) \leftarrow \mathrm{h}(x),\ \mathrm{h}(y)$$

$$\mathrm{h}(\mathrm{K}) \leftarrow$$

$$\mathrm{h}(\mathrm{N}) \leftarrow$$

$$\mathrm{h}(\mathrm{O}) \leftarrow$$

$$\dfrac{\dfrac{\mathrm{o}(x) \leftarrow \mathrm{a}(x)}{\mathrm{o}(\mathrm{K}) \leftarrow \mathrm{a}(\mathrm{K})} \qquad \dfrac{\dfrac{\mathrm{a}(x) \leftarrow \mathrm{h}(x)}{\mathrm{a}(\mathrm{K}) \leftarrow \mathrm{h}(\mathrm{K})} \qquad \mathrm{h}(\mathrm{K}) \leftarrow}{\mathrm{a}(\mathrm{K}) \leftarrow}}{\mathrm{o}(\mathrm{K}) \leftarrow}$$

Part of Speech and Structure

$\text{sentence}(xy) \leftarrow \text{noun}(x), \text{vi}(y)$

$\text{sentence}(xyz) \leftarrow \text{noun}(x), \text{vt}(y), \text{noun}(z),$

$\text{noun}(\text{K}) \leftarrow$

$\text{noun}(\text{N}) \leftarrow$

$\text{vi}(\text{walk}) \leftarrow$

$\text{vi}(\text{run}) \leftarrow$

$\text{vt}(\text{meet}) \leftarrow$

$\text{vt}(\text{love}) \leftarrow$

Example of Proof

$$\text{sentence}(xy) \leftarrow \text{noun}(x), \text{vi}(y)$$

$$\text{sentence}(xyz) \leftarrow \text{noun}(x), \text{vt}(y), \text{noun}(z),$$

$\text{noun}(K) \leftarrow$

$\text{noun}(N) \leftarrow$

$\text{vi}(\text{walk}) \leftarrow$

$\text{vi}(\text{run}) \leftarrow$

$\text{vt}(\text{meet}) \leftarrow$

$\text{vt}(\text{love}) \leftarrow$

$$\dfrac{\dfrac{\dfrac{s(xy) \leftarrow n(x), \text{vi}(y)}{s(Kr) \leftarrow n(K), \text{vi}(r)} \qquad n(K) \leftarrow}{s(Kr) \leftarrow \text{vi}(r)} \qquad \text{vi}(r) \leftarrow}{s(Kr) \leftarrow}$$

Example of Proof

sentence(xy) ← noun(x), vi(y)

sentence(xyz) ← noun(x), vt(y), noun(z),

noun(K) ←

noun(N) ←

vi(walk) ←

vi(run) ←

vt(meet) ←

vt(love) ←

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{s}(xyz) \leftarrow \text{n}(x), \text{vt}(y), \text{n}(z)}{\text{s(KmN)} \leftarrow \text{n(K), vt(m), n(N)} \qquad \text{n(K)} \leftarrow}}{\text{s(KmN)} \leftarrow \text{vt(m), n(N)} \qquad \text{vt(m)} \leftarrow}}{\text{s(KmN)} \leftarrow \text{n(N)} \qquad \text{n(N)} \leftarrow}}{\text{s(KmN)} \leftarrow}$$

Note

- Smullyan himself never consider about the structure of string data.
  - He invented EFS for his own theory of computation mechanism.
- He also used the notation

$$q_1(\tau_{11},\ldots)\rightarrow(q_2(\tau_{21},\ldots)\rightarrow(\ldots\rightarrow(q_k(\tau_{k1},\ldots)\rightarrow p(\pi_1,\ldots))\ldots)$$

  - The notation

    $$p(\pi_1,\ldots,\pi_n)\leftarrow q_1(\tau_{11},\ldots),q_2(\tau_{21},\ldots),\ldots,q_k(\tau_{k1},\ldots)$$

    was introduced later.