Computational Learning Theory
Frequent Item Set Mining

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LEARNING FROM BIT VECTORS

Data Structure : Bit Vector

- An *n*-dimension bit vector is just a sequence composed of *n* bits where a bit is from {0, 1}.

Example: (0, 0, 1, 1, 0, 0, 1, 1)

  - In the last part of this course, we assume the length of sequences in data set should be fixed and equal to *n*.
  - Sometimes each dimension of vectors is indicated with a specific name called an attribute.

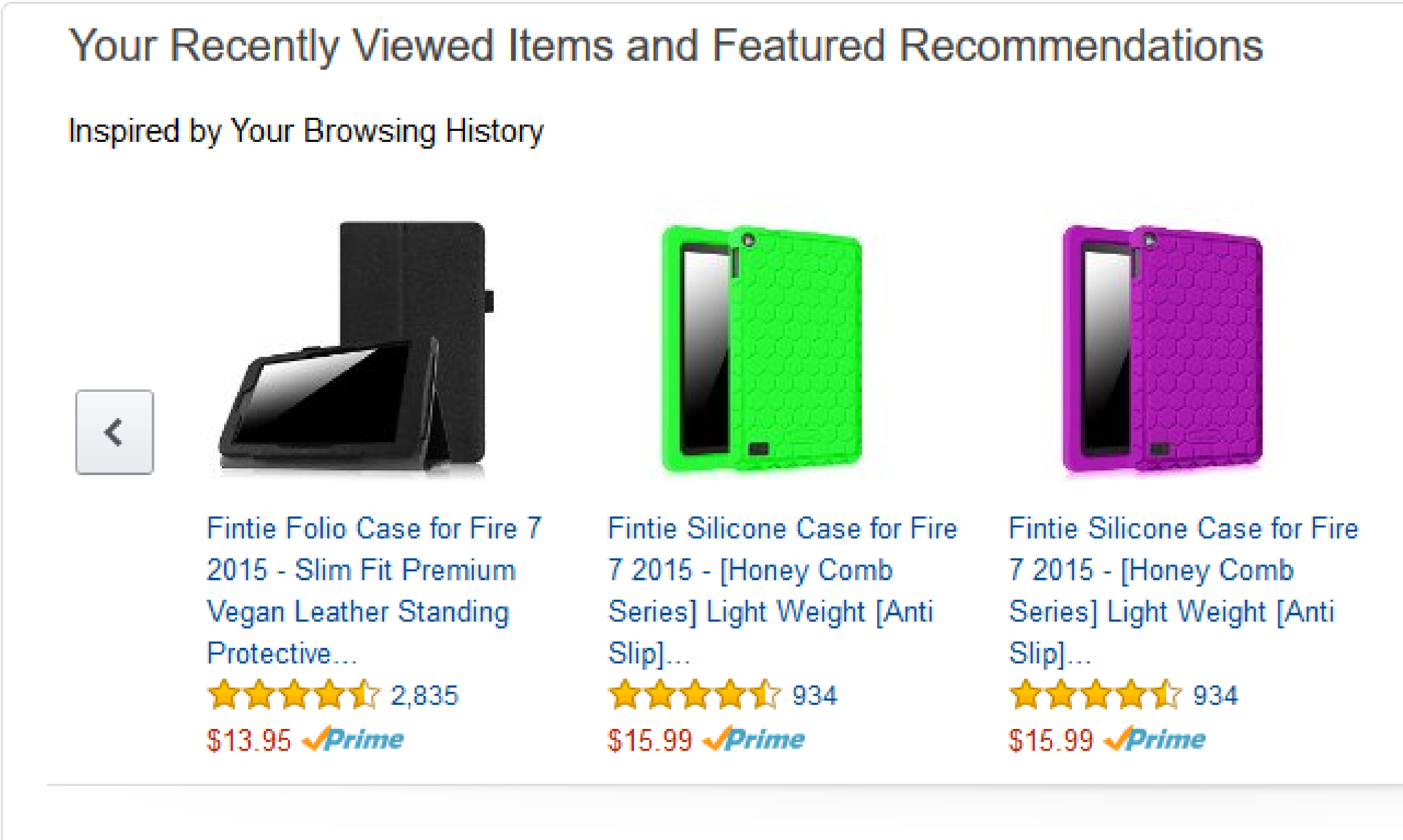
ID	A	B	C	D	E	F
1	1	0	1	1	0	0
2	0	1	1	0	1	0
3	1	1	1	0	1	0
4	1	1	0	0	1	1

ITEM SET MINING

What is item set mining?

- Originally from market basket analysis or affinity analysis
  - Market basket analysis might tell a retailer that customers often purchase shampoo and conditioner together. [Wikipedia]
- Discovering co-occurrence relationships among activities performed by (or recorded about) specific individuals or groups [Wikipedia]

For Reccomendations



A Simple Example

- Set of all items: $X = \{A, B, C, D, E, F\}$

Transaction ID	Item Sets
…	
3256	{A, C, D}
3257	{B, C, E}
3258	{A, B, C, E}
3259	{A, B, E, F}
…	….

- "Items A and C might be bought together."

Bit-vector Representation

- Every transaction can be represented as a bit-vector of n dimension, where $n = |X|$.

ID	A	B	C	D	E	F
…						
3256	1	0	1	1	0	0
3257	0	1	1	0	1	0
3258	1	1	1	0	1	0
3259	1	1	0	0	1	1
…						

Bag of Words

- Let $X = \{A_1, A_2, \ldots, A_k)$ be a finite set of words.
- For a sentence s, we define $T(s) = (x_1, x_2, \ldots, x_k)$ where

$$x_i = 1 \text{ if word } A_i \text{ appears in } s$$
$$= 0 \text{ o.w.}$$

for $i = 1,2, \ldots, n$

Example

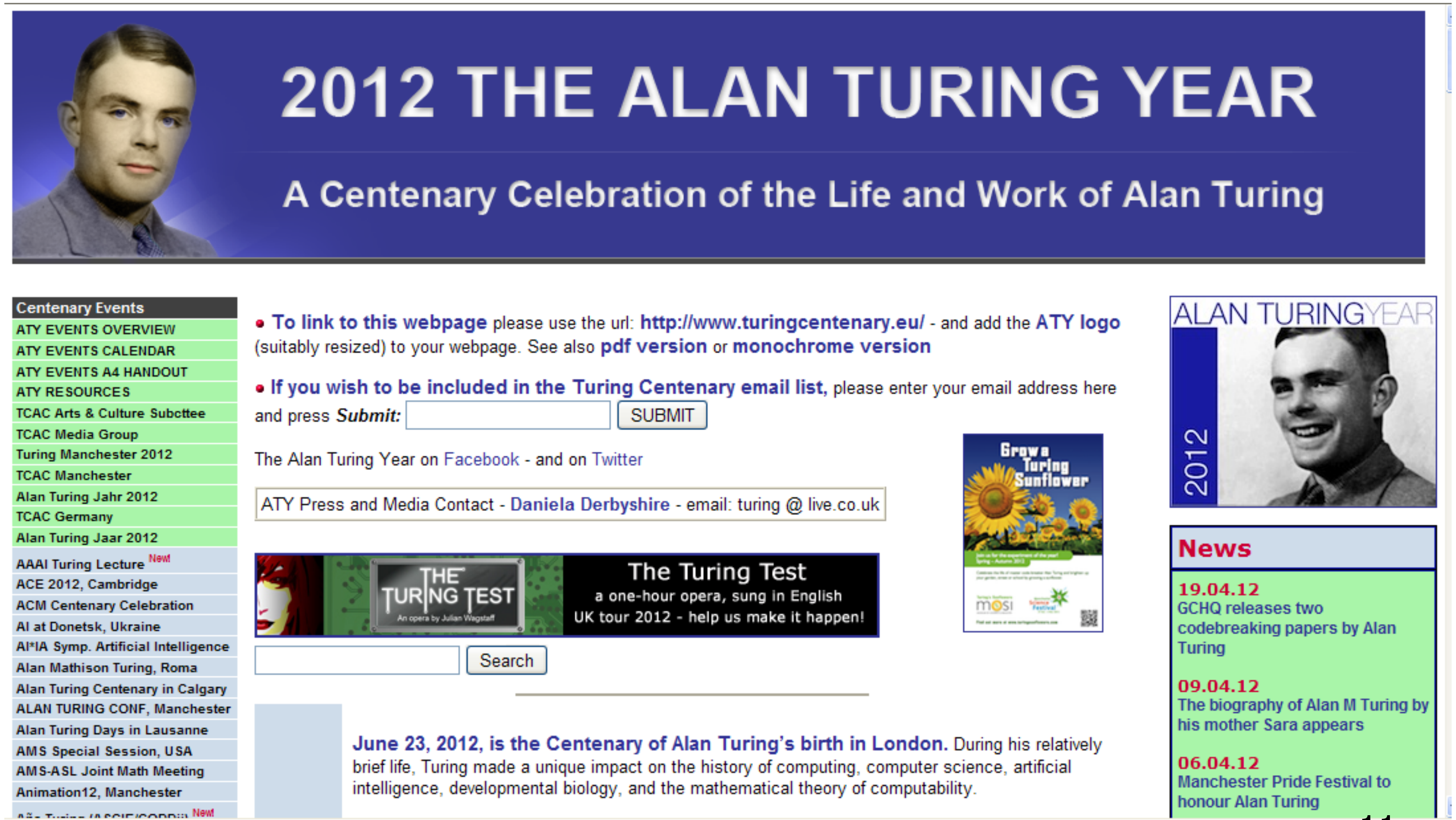
W =(arithmetic, book, compute, paper, suppose, square, symbol, write)

s_1: Computing is normally done by writing certain symbols on paper.

s_2: We may suppose this paper is divided into squares like a child's arithmetic book.

$$T(s_1) = (0, 0, 1, 1, 0, 0, 1, 1)$$
$$T(s_2) = (1, 1, 0, 1, 1, 1, 0, 0)$$

- Alan Turing: On Computable Numbers, with an Application to the Entscheidungsproblem: A correction". Proceedings of the London Mathematical Society 43: pp. 544–6. 1937. doi:10.1112/plms/s2-43.6.544



Mathematical Definitions

- Assuming a finite set of all items as attributes

$$X = \{A_1, A_2, \ldots, A_n\}$$

- A transaction is a pair $t = (i, T)$ of an identifier $i \in \mathbf{N}$ and a finite set of items $T \in X$
- A transaction database D is a finite set of transactions in which no pair of transactions have a same identifier, that is,

  $t = (i, T) \in D$ and $s = (j, S) \in D$ imply $i \neq j$.
- A pattern is a finite set of items.
  - Transactions are for training data patterns are rules.

Mathematical Definitions (2)

- For a pattern P and a transaction $t = (i, T)$, we say t satisfies P (or P matches t) iff $P \subset T$.
- Let $D(P) = \{ t \mid P \text{ matches } t \}$.
- The support of P in a transaction database D is defined as supp(P) $= | D(P) | / | D |$.
  - The support is also called the relative frequency.

Definition of Learning Task

- Assuming a set of items X
- For a given transaction database D and a minimal support (threshold) σ s.t. $0 \leq \sigma \leq 1$, enumerate all patterns P s.t. supp(P) $\geq \sigma$.

A Very Simple Example

ID	A	B	C	D	E	F
1	1	0	1	1	0	0
2	0	1	1	0	1	0
3	1	1	1	0	1	0
4	1	1	0	0	1	1

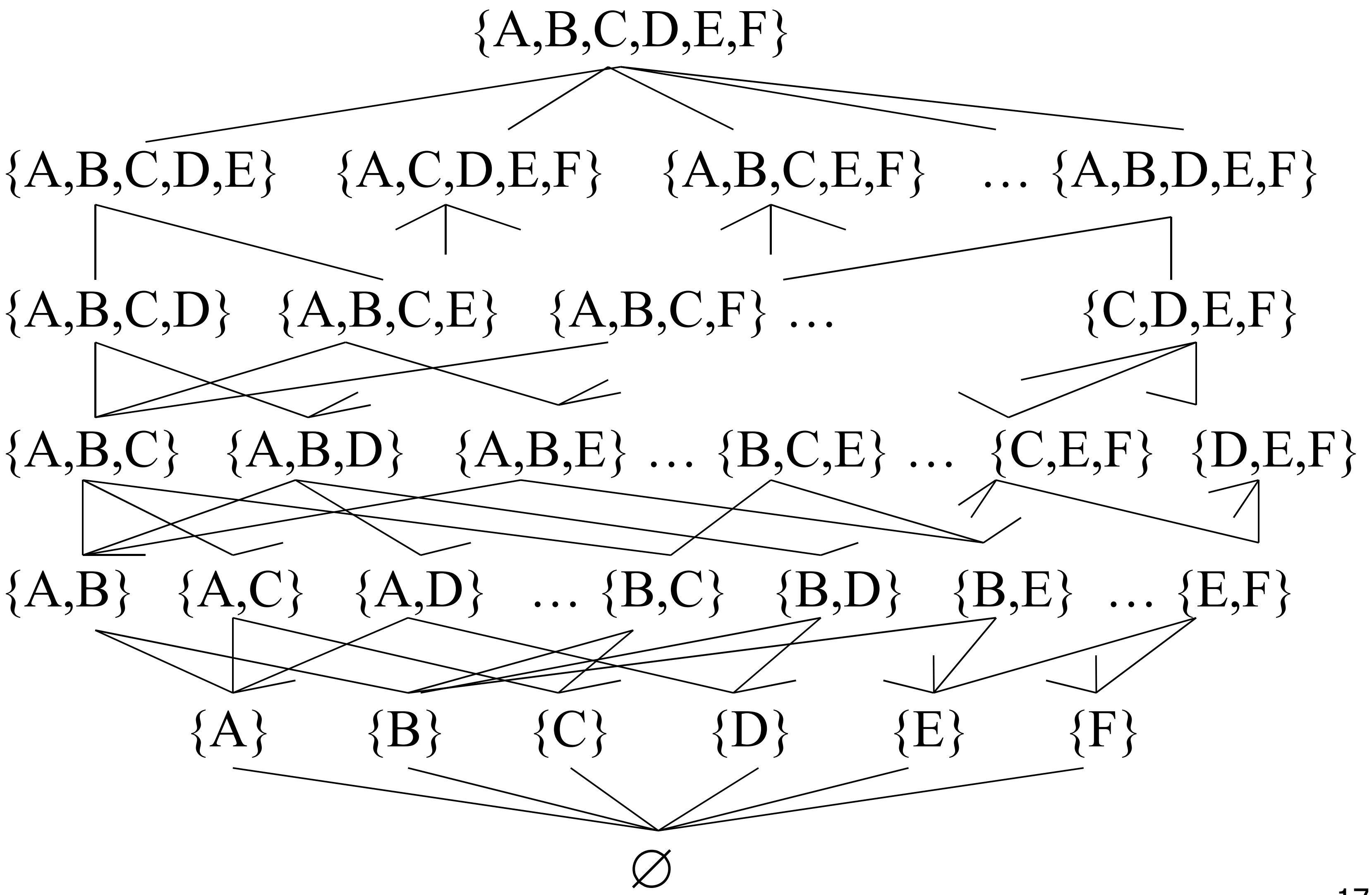
supp({A})= supp({B})= supp({C})= supp({E})= 0.75,

supp({D})= supp({F})= 0.25

supp({A, B})=supp({A, C})=0.5, supp({A, D})=0.25,…

THE A-PRIORI ALGORITHM

Hasse Diagram of Patterns



Monotonicity of the Support

Lemma For two patterns P and Q,

$$P \subseteq Q \Rightarrow \text{supp}(P) \geq \text{supp}(Q)$$

ID	A	B	C	D	E	F
1	1	0	1	1	0	0
2	0	1	1	0	1	0
3	1	1	1	0	1	0
4	1	1	0	0	1	1

supp({A})=0.75 ≥ supp({A , B})=0.25
supp({B})=0.5≥ supp({A , B})=0.25
supp({A})= 0.75 ≥ supp({A , C})=0.5

A Propri Algorithm [Agrawal et al. 93]

1. Let $k = 1$.
2. Let $C_1 = \{ \{A\} \mid A \in X \}$.
3. Let $L_k = \{ P \in C_k \mid \mathrm{supp}(P) \geq \sigma \}$.
4. If $L_k = \emptyset$ then halt, otherwise

   Let $C_{k+1} = \{ P \cup Q \mid P \in L_k, Q \in L_k, |P \cup Q| = k+1,$

   but $P \cup Q$ does not subsumes any

   $R \in C_i - L_i \ (i \leq k) \}$.

   Increment k.

   Repeat Step 4.

An Example of Run(1)

ID	A	B	C	D	E	F
1	1	0	1	1	0	0
2	0	1	1	0	1	0
3	1	1	1	0	1	0
4	1	1	0	0	1	1

$\sigma = 0.5$

$C_1 = \{\{A\}, \{B\}, \ldots, \{F\}\}$

$L_1 = \{\{A\},\{B\},\{C\},\{E\}\}$

$C_2 = \{\{A, B\}, \{A, C\}, \{A, E\}, \{B, C\}, \{B, E\}, \{C, E\}\}$

$L_2 = \{\{A, B\},\{A, C\}, \{A, E\}, \{B, C\}, \{B, E\}, \{C, E\}\}$

$C_3 = \{\{A,B,C\}, \{A,B,E\} \{B,C,E\}\}$

$L_3 = \{\{A,B,E\},\{B,C,E\}\}$

An Example of Run (2)

ID	A	B	C	D	E	F
1	1	0	1	1	0	0
2	0	1	1	0	1	0
3	1	1	1	0	1	0
4	0	1	0	0	1	1

$\sigma = 0.5$

$C_1 = \{\{A\}, \{B\}, \ldots, \{F\}\}$

$L_1 = \{\{A\},\{B\},\{C\},\{E\}\}$

$C_2 = \{$~~{A, B},~~ $\{A, C\},$
~~{A, E},~~ $\{B, C\},$
$\{B, E\}, \{C, E\}\}$

$L_2 = \{\{A, C\}, \{B,C\},$
$\{B, E\}, \{C, E\}\}$

$C_3 = \{$~~{A,B,C}~~, ~~{A,B,E}~~
$\{B,C,E\}\}$

$L_3 = \{B,C,E\}$

An Example of Run(3)

ID	A	B	C	D	E	F
1	1	1	0	1	0	0
2	0	1	1	0	1	0
3	1	1	1	0	1	0
4	0	1	0	0	1	1

$\sigma = 0.5$

$C_1 = \{\{A\}, \{B\}, \ldots, \{F\}\}$

$L_1 = \{\{A\},\{B\},\{C\},\{E\}\}$

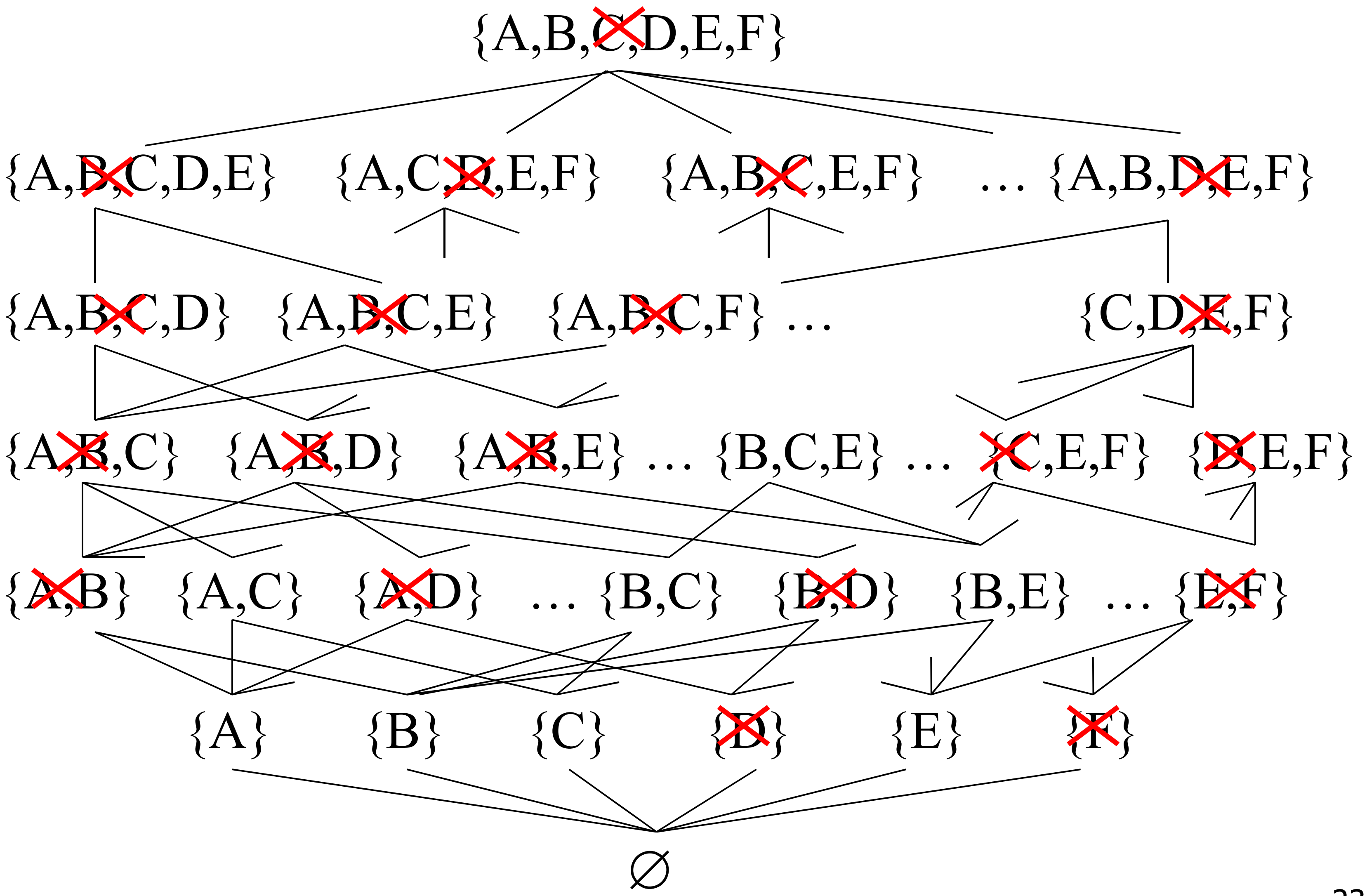
$C_2 = \{\{A, B\}, \sout{\{A, C\}}, \sout{\{A, E\}}, \{B, C\}, \{B, E\}, \{C, E\}\}$

$L_2 = \{\{A, B\}, \{B, C\}, \{B, E\}, \{C, E\},\}$

$C_3 = \{\sout{\{A, B, C\}}, \sout{\{A, B, E\}}, \sout{\{A, C, E\}}, \{B, C, E\}\}$

$L_3 = \{\{B, C, E\}\}$

Hasse Diagram of Patterns



DEPTH-FIRST SEARCH

A Propri Algorithm [Agrawal et al. 93]

1. Let $k = 1$.
2. Let $C_1 = \{\{A\} \mid A \in X\}$.
3. Let $L_k = \{P \in C_k \mid \text{supp}(P) \geq \sigma\}$.
4. If $L_k = \emptyset$ then halt, otherwise

   Let $C_{k+1} = \{\, P \cup Q \mid P \in L_k, Q \in L_k, |P \cup Q| = k+1,$ but $P \cup Q$ does not subsumes any $R \in C_i - L_i \, (i \leq k) \,\}$.

   Increment k.

   Repeat Step 4.

Depth-First Search Algorithm(1)

- Assuming a total ordering for the set X of items.

  Example : A > B > C > D > E > F

- Regarding (Implementing) every pattern $P \in L_k$ as a list of items in which items are ordered in the descending order.

↓

For two lists $P = [P', A_i] \in L_k$ and $Q = [P', A_j] \in L_k$ of descending order, the list $[P', A_i, A_j]$ does not subsume any $R \in C_i - L_i \, (i \leq k)$.

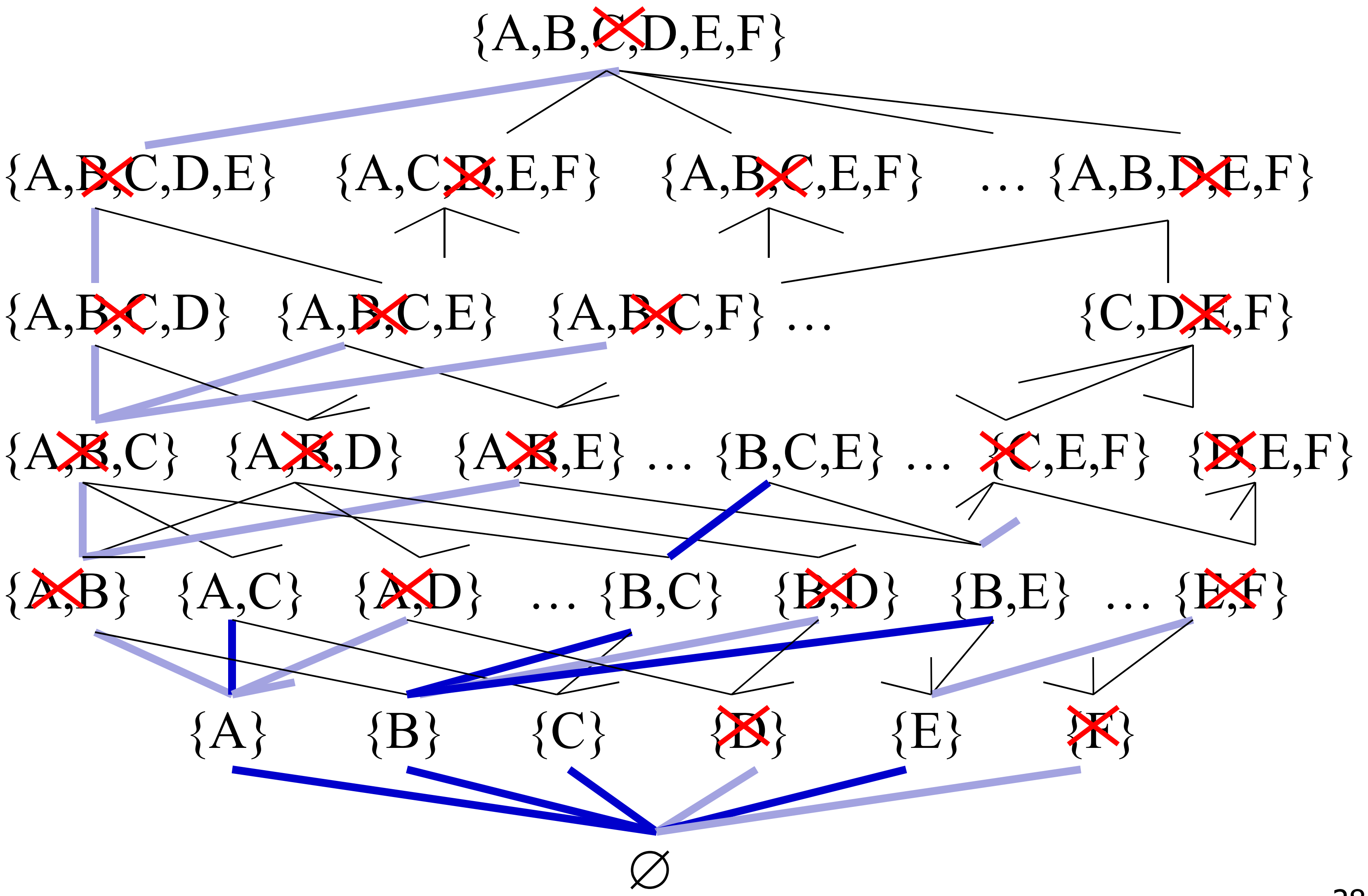
Depth-First Search Algorithm(2)

- A more simplified method is to let

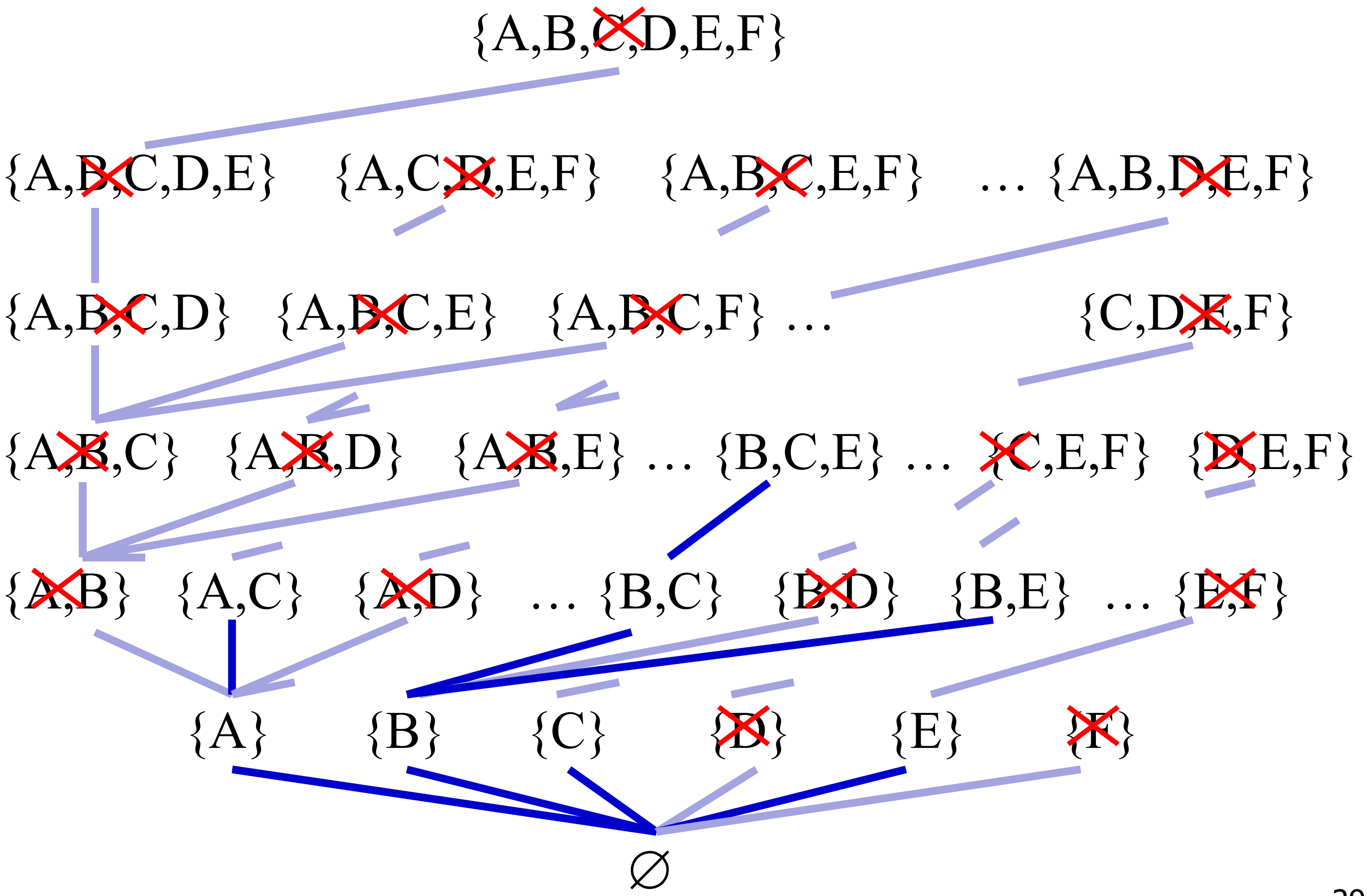
$$C_{k+1} = \{ [P, A_i, A_j] \mid [P, A_i] \in L_k \text{ and } A_i > A_j \}$$

- Instead of this version of C_{k+1} as it is, we can design a depth-first search algorothm.

Depth-First Search in the Diagram



Depth-First Search in the Diagram

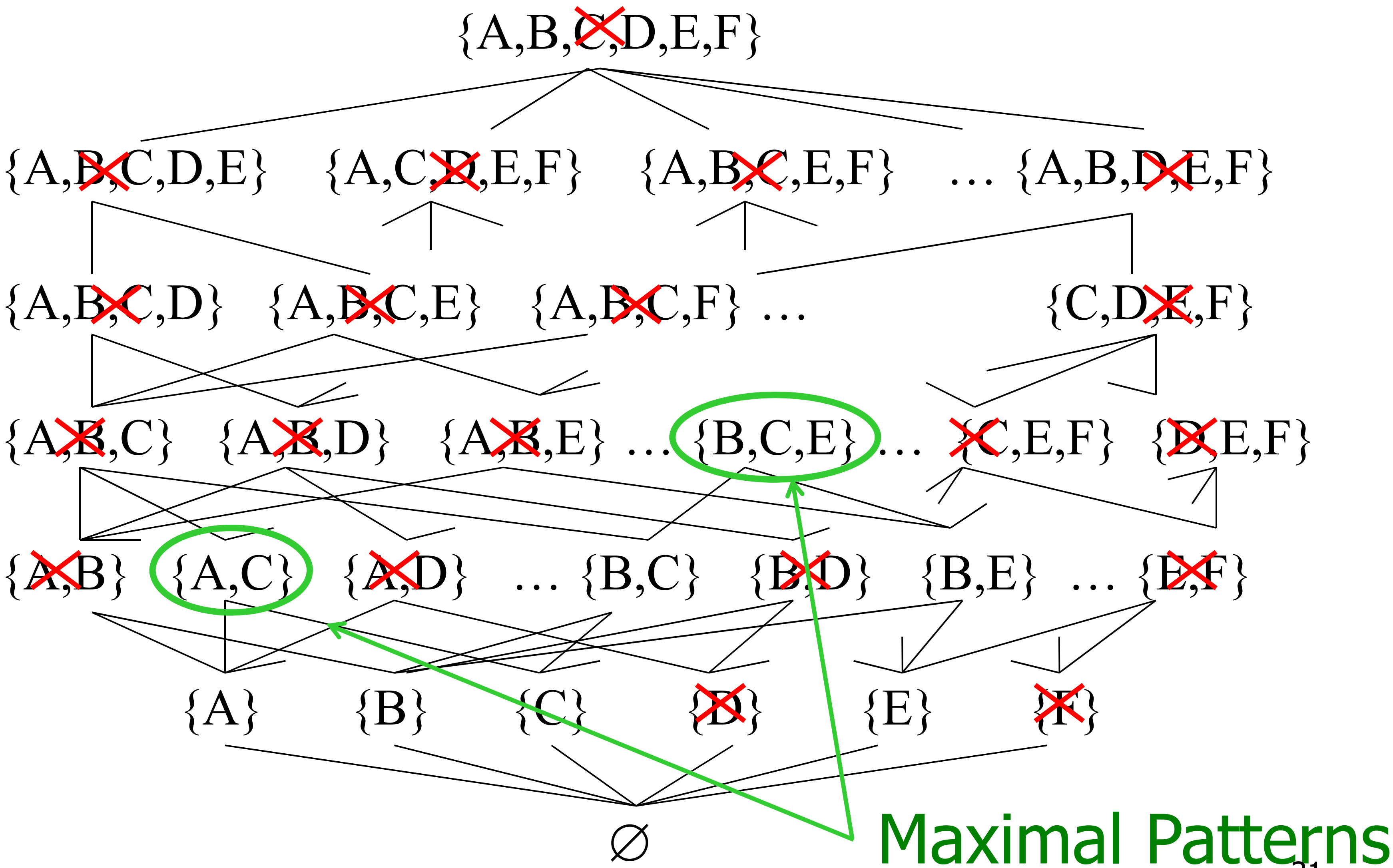


Maximal Patterns

- A pattern P is maximal as the answer of the task if
  $\mathrm{supp}(P) \geq \sigma$ and
  no pattern Q s.t. $Q \supset P$ satisfies $\mathrm{supp}(Q) \geq \sigma$.

- From the monotonicity of the support function, every subset S of a maximal pattern P satisfies
  $\mathrm{supp}(S) \geq \sigma$.
  - We may enumerate only maximal patterns.

Maximal Patterns in the Hasse Diagram



FP-TREES

What is an FP-Tree?

- We regard (implement) every transaction as a list of items in the descending order defined by the support of each item.
  - When a minimal support σ is given, we can neglect all items A such that supp(A) < σ.
- We regard (implement) a transaction database D as a prefix tree $T'(D)$.
- An FP-tree $T(D)$ is obtained by giving links among nodes whose labels are same.

Example of FP-tree(1)

ID	Item Set
1	{A, B, D}
2	{B, C, E}
3	{A, B, C, E}
4	{B, E, F}

ID	A	B	C	D	E	F
1	1	1	0	1	0	0
2	0	1	1	0	1	0
3	1	1	1	0	1	0
4	0	1	0	0	1	1

- Constructing the table of the supports

  $\sigma = 0.5$

B	4	
E	3	
A	2	
C	2	
~~D~~	~~1~~	
~~E~~	~~1~~	

Example of FP-tree(2)

- Represent every transaction as a list of the descending order.

B	4	
E	3	
A	2	
C	2	

ID	Item Set
1	{A, B, D}
2	{B, C, E}
3	{A, B, C, E}
4	{B, E, F}

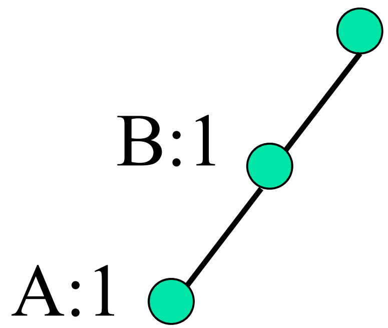
ID	Item List
1	[B, A, ~~D~~]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E, ~~F~~]

Example of FP-tree(3)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

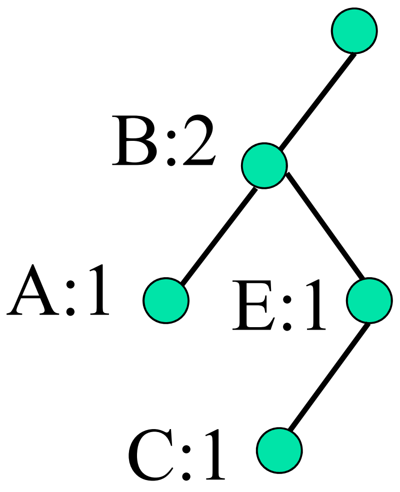
$\sigma = 0.5$

B	4	
E	3	
A	2	
C	2	



Example of FP-tree(4)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]



$\sigma = 0.5$

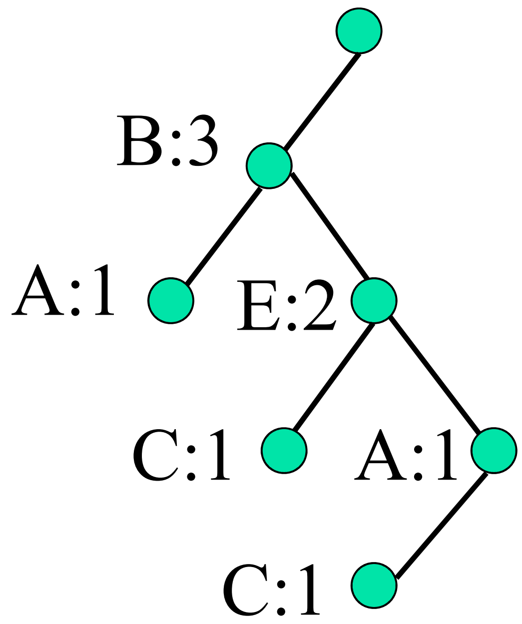
B	4	
E	3	
A	2	
C	2	

Example of FP-tree(5)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$

B	4	
E	3	
A	2	
C	2	

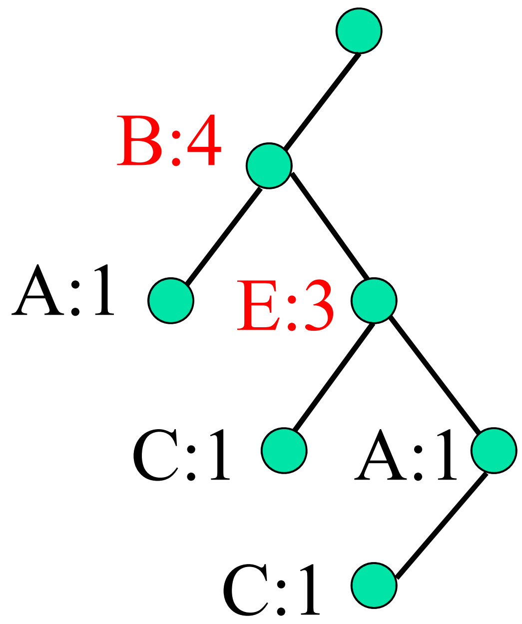


Example of FP-tree(6)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$

B	4	
E	3	
A	2	
C	2	

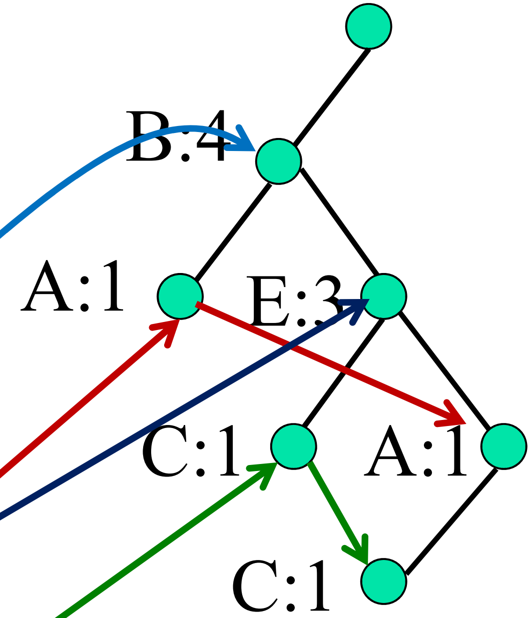


Example of FP-tree(7)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$

B	4	
E	3	
A	2	
C	2	



The FP-Growth Algorithm[Han et al. 00]

- Given a minimal support σ
- Let L be the list of items $[A_1, A_2 \ldots, A_m]$ satisfying $\text{supp}(A_k) \geq \sigma$ in the ascending order of the support.

FP-growth(T, L)

1. If T consists of one path p, enumerate all patterns at least one A_i s.t. $\text{supp}_N(A_i) \geq \sigma$ and all items in L.
2. For $k = 1,2, \ldots, n$, repeat the following:

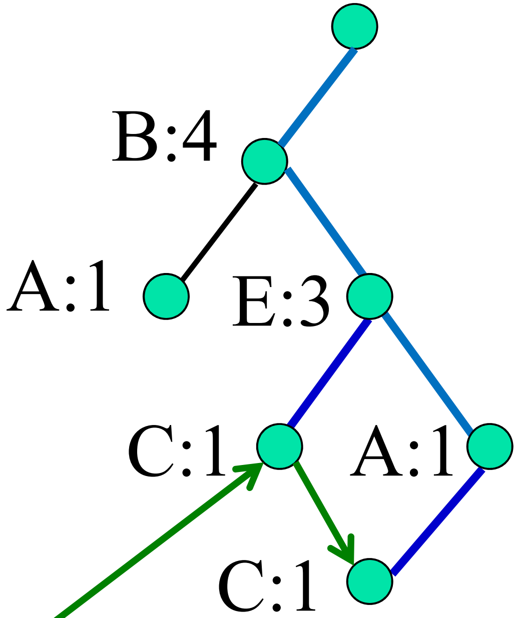
- Construct the conditional transaction database D' and FP-tree $T(D')$ by gathering items from the root of T and the parent of A_k, and execute FP-growth(T', $[A_k , L]$).

Example Run of FP-Growth(1-1)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$

B	4	
E	3	
A	2	
C	2	



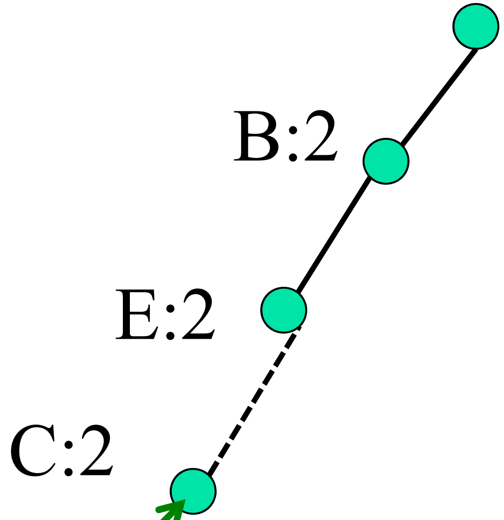
Conditional Data

Item List	supp
[B, E, C]	1
[B, E, A, C]	1

Example Run of FP-Growth(1-2)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$



B	2	
E	2	
C	2	

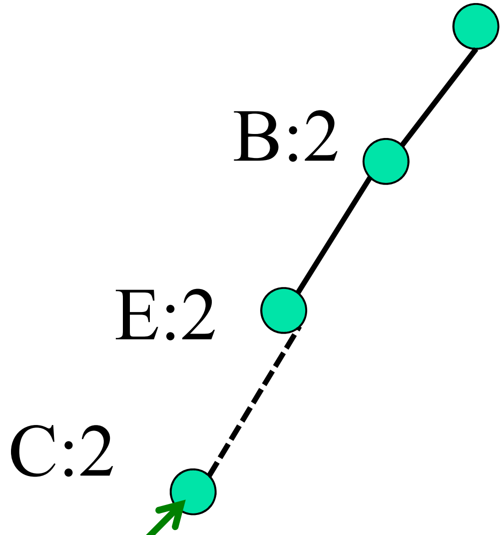
Conditional Data

Item List	supp
[B, E, C]	1
[B, E, A, C]	1

Example Run of FP-Growth(1-3)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$



B	2	
E	2	
C	2	

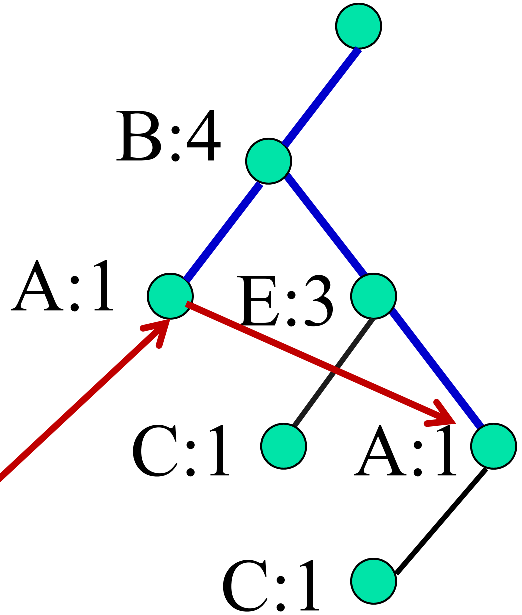
supp({ B, E, C}) =0.5
supp({ B, C }) =0.5
supp({ E, C }) =0.5
supp({ C }) =0.5

Example Run of FP-Growth(1-4)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$

B	4	
E	3	
A	2	
C	2	



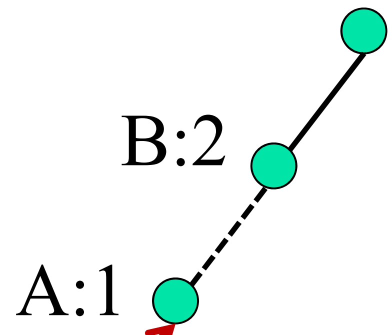
Conditional Data

Item List	supp
[B, A]	1
[B, E, A]	1

Example Run of FP-Growth(1-5)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$



B	2	
A	2	

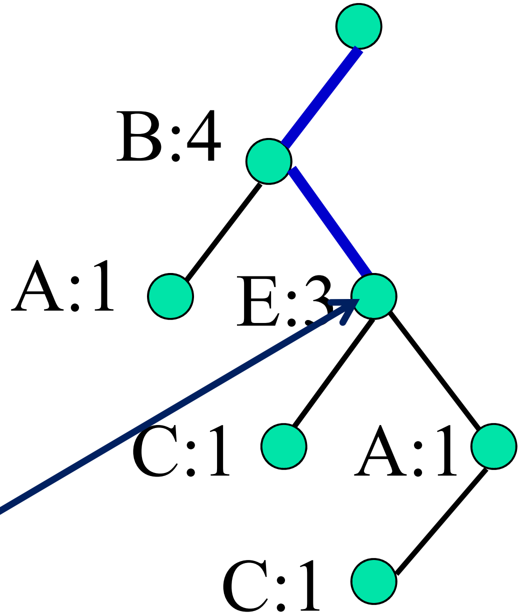
supp({B, A})=0.5
supp({A })=0.5

Example Run of FP-Growth(1-6)

ID	A	B	C	D	E	F
1	1	1	0	1	0	0
2	0	1	1	0	1	0
3	1	1	1	0	1	0
4	0	1	0	0	1	1

$\sigma = 0.5$

B	4	
E	3	
A	2	
C	2	



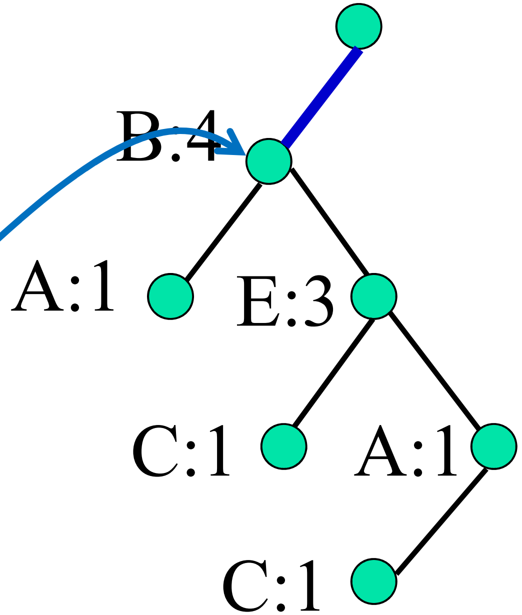
supp({ B, E })=0.75

Example Run of FP-Growth(1-7)

ID	Item List
1	[B, A]
2	[B, E, C]
3	[B, E, A, C]
4	[B, E]

$\sigma = 0.5$

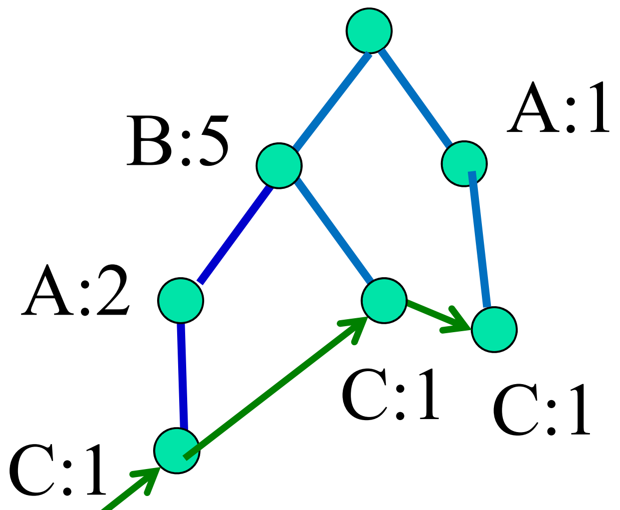
B	4	
E	3	
A	2	
C	2	



supp({B})=1

Example Run of FP-Growth(2-1)

ID	Item List
1	[B,A]
2	[B, C, D]
3	[B, A, C]
4	[B,A]
5	[A, C]
6	[B, E]



$\sigma = 0.3$

B	5	
A	4	
C	3	

Conditional Data

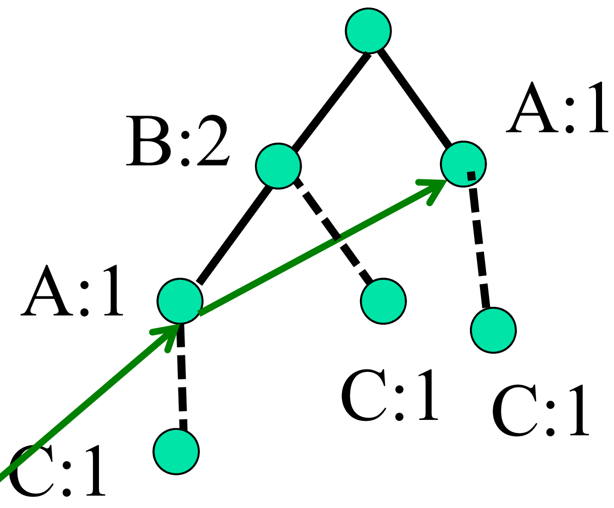
Item List	supp
[B, A, C]	1
[B, C]	1
[A, C]	1

Example Run of FP-Growth(2-2)

ID	Item List
1	[B,A]
2	[B, C, D]
3	[B, A, C]
4	[B,A]
5	[A, C]
6	[B, E]

$\sigma = 0.3$

B	2	
A	2	

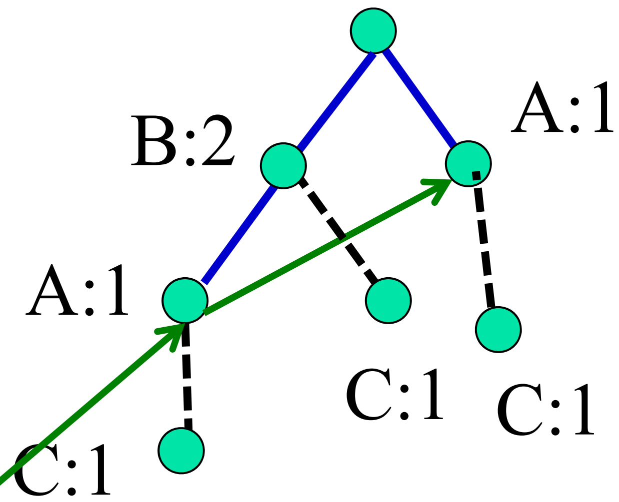


Conditional Data

Item List	supp
[B, A, C]	1
[B, C]	1
[A, C]	1

Example Run of FP-Growth(2-3)

ID	Item List
1	[B,A]
2	[B, C, D]
3	[B, A, C]
4	[B,A]
5	[A, C]
6	[B, E]



$\sigma = 0.3$

B	2	
A	2	

Conditional Data

Item List	supp
[B, A, C]	1
[A, C]	1

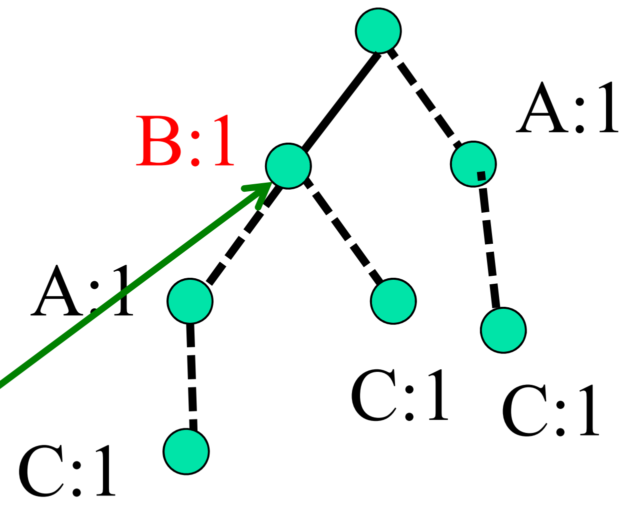
Example Run of FP-Growth(2-4)

ID	Item List
1	[B,A]
2	[B, C, D]
3	[B, A, C]
4	[B,A]
5	[A, C]
6	[B, E]

$\sigma = 0.3$

B	1	

supp{A, C}=0.333…



Conditional Data

Item List	supp
[B, A, C]	1
[A, C]	1

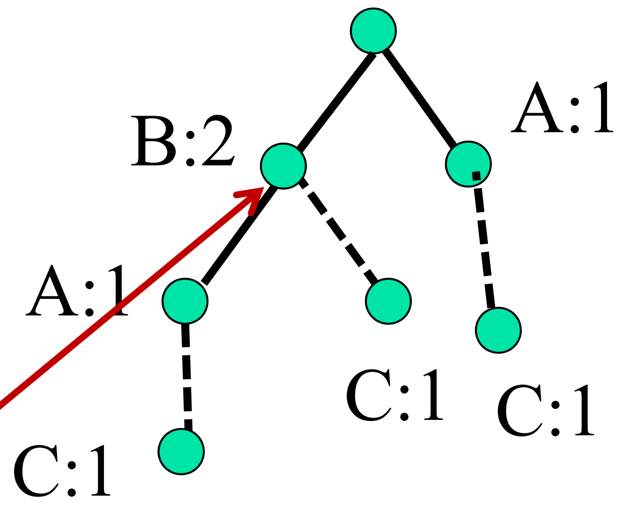
Example Run of FP-Growth(2-5)

ID	Item List
1	[B,A]
2	[B, C, D]
3	[B, A, C]
4	[B,A]
5	[A, C]
6	[B, E]

$\sigma = 0.3$

B	2	
A	2	

supp{B, C}=0.333…
supp{C}=0.333…



Conditional Data

Item List	supp
[B, A, C]	1
[B, C]	1
[A, C]	1